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On the positive weak almost limited operators

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Abstract. Using the concept of approximately order bounded sets with respect to a lattice seminorm, we establish some new characterizations of positive weak almost limited operators on Banach lattices. Consequently, we derive some results about the weak Dunford–Pettis* and the Dunford–Pettis* property of σ-Dedekind complete Banach lattices.

Keywords: Weak almost limited operator; The weak Dunford–Pettis* property; Banach lattice

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1. INTRODUCTION AND NOTATIONS

Throughout this paper X, Y will denote real Banach spaces, and E, F will denote real Banach lattices. E^+ denotes the positive cone of E and $sol\,(A)$ denotes the solid hull of a subset A of a Banach lattice. The notation $x_n \perp x_m$ will mean that the sequence (x_n) of a Banach lattice is disjoint, that is, $|x_n| \wedge |x_m| = 0$, $n \neq m$. An operator $T : E \to F$ is positive if $T(x) \geq 0$ in F whenever $x \geq 0$ in E. A lattice seminorm ϱ on a Banach lattice E is a seminorm such that for every $x, y \in E$, $|x| \leq |y|$ implies $\varrho(x) \leq \varrho(y)$. The closed unit ball associated to a lattice seminorm ϱ is defined by $B_\varrho = \{x \in E : \varrho(x) \leq 1\}$. The lattice operations in a Banach lattice E (resp. E') are weakly (resp. weak*) sequentially continuous if for every weakly null sequence (x_n) in E (resp. weak* null sequence (f_n) in E'), $|x_n| \to 0$ for $\sigma(E, E')$ (resp. $|f_n| \to 0$ for $\sigma(E', E)$). Finally, we will use the term

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operator $T : E \to F$ between two Banach lattices to mean a bounded linear mapping. We refer to [1,6] for unexplained terminology of Banach lattice theory and positive operators.

Several types of the Dunford–Pettis property are considered in the theory of Banach lattices. Namely, a Banach lattice E has

- the *Dunford–Pettis property*, whenever $x_n \overset{w}{\to} 0$ in E and $f_n \overset{w}{\to} 0$ in E' imply $f_n(x_n) \to 0$.
- the *Dunford–Pettis* property*, whenever $x_n \overset{w}{\to} 0$ in E and $f_n \overset{w^*}{\to} 0$ in E' imply $f_n(x_n) \to 0$.
- the *weak Dunford–Pettis property* (abb. wDP property) [7], whenever

$$x_n \perp x_m,\ x_n \overset{w}{\to} 0 \text{ in } E \text{ and } f_n \overset{w}{\to} 0 \text{ in } E' \text{ imply } f_n(x_n) \to 0.$$

- the *weak Dunford–Pettis** property (abb. wDP* property), whenever

$$x_n \overset{w}{\to} 0 \text{ in } E \text{ and } f_n \perp f_m, f_n \overset{w^*}{\to} 0 \text{ in } E' \text{ imply } f_n(x_n) \to 0.$$

The wDP* property, introduced recently by J. X. Chen et al. [3], is a weak version of the Dunford–Pettis* property and stronger than the wDP property. Note that the weak Dunford–Pettis* property is related to the so called *weak almost limited* operators. An operator $T : E \to F$ between Banach lattices is said to be weak almost limited [4], whenever

$$x_n \overset{w}{\to} 0 \text{ in } E \text{ and } f_n \perp f_m, f_n \overset{w^*}{\to} 0 \text{ in } F' \text{ imply } f_n(T(x_n)) \to 0.$$

Clearly, a Banach lattice E has the weak Dunford–Pettis* property if and only if the identity operator on E is weak almost limited.

Let us recall that an operator $T : X \to Y$ is said to be *limited* if $\|T^*(f_n)\| \to 0$ for every weak* null sequence $(f_n) \subset Y^*$. Furthermore, An operator $T : X \to E$ from a Banach space into a Banach lattice is said to be *almost limited* [5], if $\|T^*(f_n)\| \to 0$ for every disjoint weak* null sequence $(f_n) \subset E^*$. Accordingly, a Banach lattice E is said to have the *Schur property* (resp. *dual Schur property* [5]), if weakly null sequences in E are norm null (resp. disjoint weak* null sequences in E' are norm null). For a σ-Dedekind complete Banach lattice E (see [5, Theorem 3.3]), the dual Schur property coincide with the so called *dual positive Schur property* [2], that is, weak* null sequences in $(E')^+$ are norm null. Clearly, a Banach lattice E has the dual Schur property if and only if the identity operator on E is almost limited. For an operator $T : E \to F$ between Banach lattices the following implications are clear:

$$T \text{ is limited} \Rightarrow T \text{ is almost limited} \Rightarrow T \text{ is weak almost limited.}$$

However, there is a weak almost limited operator which needs not to be almost limited (and hence limited). Indeed, the identity operator $I : \ell^1 \to \ell^1$ is weak almost limited as ℓ^1 has the Schur (wDP*) property. But, as ℓ^1 does not have the dual positive Schur property [8, Proposition 2.1], $I : \ell^1 \to \ell^1$ is not almost limited. On the other hand, the identity operator on the Banach lattice c is not weak almost limited. Indeed, let $f_n \in c^* = \ell^1$ be such that $f_n = (0, \ldots, 0, 1_{(2n)}, -1_{(2n+1)}, 0, \ldots)$. Then (f_n) is a disjoint weak* null sequence in c^* [3, Example 2.1(2)], and clearly, the sequence (x_n) defined by $x_n = (0, \ldots, 0, 1_{(2n)}, 0, \ldots) \in c$ is weakly null, but $f_n(x_n) = 1$ for all n.

In this paper, using the concept of approximately order bounded sets with respect to a lattice seminorm, we establish a characterization of positive weak almost limited operators

(Theorem 2.5), and give consequently in terms of sequences in E and F', several characterizations of positive weak almost limited operators from E into a σ-Dedekind complete Banach lattice F (Theorem 2.7). As consequences we derive some new characterizations of the wDP* property of a σ-Dedekind complete Banach lattice (Corollary 2.10). Finally, we establish some sufficient conditions under which the wDP* and the Dunford–Pettis* properties coincide (Corollary 2.12).

2. Main results

The following lemmas will be used throughout this paper.

Lemma 2.1. *Let E be a Banach lattice, let $(x_n) \subset E^+$ be a norm bounded sequence and let $x = \sum_{n=1}^{\infty} 2^{-n} x_n$. Then the sequences (u_n) and (v_n) defined for every $n \geq 2$ by*

$$u_n = \left(x_n - 2^n \sum_{i=1}^{n-1} x_i - x \right)^+$$

and

$$v_n = \left(x_n - 4^n \sum_{i=1}^{n-1} x_i - 2^{-n} x \right)^+$$

are a disjoint sequences.

Proof. Note that the proof is similar for the two sequences. If $n > m \geq 2$, then we have

$$0 \leq u_n \leq (x_n - 2^n x_m)^+ ,$$

and

$$0 \leq 2^n u_m \leq 2^n \left(x_m - 2^{-n} x_n \right)^+ = (x_n - 2^n x_m)^- .$$

So, from $(x_n - 2^n x_m)^+ \perp (x_n - 2^n x_m)^-$ we see that $u_n \perp u_m$ as desired. □

Lemma 2.2 ([1, Theorem 4.34]). *If A is a relatively weakly compact subset of a Banach lattice E, then every disjoint sequence in the solid hull of A converges weakly to zero. In particular, for every sequences (x_n), $(y_n) \subset E$ such that $|y_n| \leq |x_n|$, $y_n \perp y_m$ and $x_n \xrightarrow{w} 0$ we have $y_n \xrightarrow{w} 0$.*

Lemma 2.3 ([3, Lemma 2.2]). *Let E be a σ-Dedekind complete Banach lattice. Then for every sequences (f_n), $(g_n) \subset E'$ such that $|g_n| \leq |f_n|$, $g_n \perp g_m$ and $f_n \xrightarrow{w^*} 0$ we have $g_n \xrightarrow{w^*} 0$.*

Let us recall that for a lattice seminorm ϱ on a Banach lattice E, a subset A of E is said to be *approximately order bounded* with respect to ϱ if for every $\varepsilon > 0$ there exists $u \in E^+$ such that $A \subset [-u, u] + \varepsilon B_\varrho$ (see [6, Remark, p. 73]). Note that from [6, Remark, p. 73], it follows that $A \subset E$ is approximately order bounded with respect to ϱ if and only if for every $\varepsilon > 0$ there exists $u \in E^+$ such that $\varrho\left((|x| - u)^+\right) \leq \varepsilon$ for every $x \in A$. Moreover, if $A \subset E$ is a norm bounded subset, and $T : E \to F$ is a positive operator, then it is easy to see that $\varrho_{T,A}(f) := \sup\{|f|(T(|x|)) : x \in A\}$ defines a lattice seminorm on F'. For the identity operator $I : E \to E$, we get the lattice seminorm on E' defined by $\varrho_A(f) = \sup\{|f|(|x|) : x \in A\}$.

We shall need the following lemma which characterizes approximately order bounded sequences with respect to a lattice seminorm.

Lemma 2.4. *A sequence (x_n) of a Banach lattice E is approximately order bounded with respect to a lattice seminorm ϱ, if and only if for every $\varepsilon > 0$ there exist $u \in E^+$ and a natural number k such that $\varrho\left((|x_n| - u)^+\right) \leq \varepsilon$ for every $n > k$.*

Proof. The "only if" part is obvious. For the "if" part, let $\varepsilon > 0$. There exist $u \in E^+$ and a natural number k such that $\varrho\left((|x_n| - u)^+\right) \leq \varepsilon$ for every $n > k$. Put $v_k = \bigvee_{n=1}^{k} |x_n|$ and $v = u + v_k$. So $\varrho\left((|x_n| - v)^+\right) \leq \varepsilon$ holds for every n. In fact,

- if $n \leq k$ then $\varrho\left((|x_n| - v)^+\right) = \varrho(0) = 0 \leq \varepsilon$;
- if $n > k$ then $(|x_n| - v)^+ \leq (|x_n| - u)^+$ and hence

$$\varrho\left((|x_n| - v)^+\right) \leq \varrho\left((|x_n| - u)^+\right) \leq \varepsilon.$$

This ends the proof. □

Our following result characterizes positive weak almost limited operators from E into σ-Dedekind complete Banach lattice F through weak* null sequences in F' that are approximately order bounded with respect to a lattice seminorm.

Theorem 2.5. *Let E and F be two Banach lattices such that F is σ-Dedekind complete. Then, a positive operator $T : E \to F$ is a weak almost limited if, and only if, each weak* null sequence $(f_n) \subset F'$ is approximately order bounded with respect to the lattice seminorm $\varrho_{T,A}$ for every relatively weakly compact set $A \subset E$.*

Proof. For the "only if" part, assume by way of contradiction that there exist a weak* null sequence $(f_n) \subset F'$, a relatively weakly compact subset $A \subset E$, such that (f_n) is not approximately order bounded with respect to $\varrho_{T,A}$. That is by Lemma 2.4, there is some $\varepsilon > 0$ so that for each $g \in (F')^+$ and each natural number k we have

$$\varrho_{T,A}\left((|f_n| - g)^+\right) > \varepsilon$$

for at least one $n > k$ and thus, $(|f_n| - g)^+ (T|x_n|) > \varepsilon$ for at least one $x_n \in A$. In particular, an easy inductive argument shows that there exist a subsequence of (f_n) (which we still denote (f_n)) and a sequence $(x_n) \subset A$ such that

$$\left(|f_n| - 4^n \sum_{i=1}^{n-1} |f_i|\right)^+ (T|x_n|) > \varepsilon$$

holds for all $n \geq 2$. Let $f = \sum_{n=1}^{\infty} 2^{-n} |f_n|$ and

$g_n = \left(|f_n| - 4^n \sum_{i=1}^{n-1} |f_i| - 2^{-n} f\right)^+$ $(n \geq 2)$. Clearly, $0 \leq g_n \leq |f_n|$ holds for every n, and note that from Lemma 2.1 (g_n) is a disjoint sequence. Then by Lemma 2.3, $g_n \xrightarrow{w^*} 0$. Hence, as T is weak almost limited we see that $T(\mathrm{sol}(A))$ is an almost limited set ([4, Theorem 2.4 (5)]), and then $g_n(T|x_n|) \to 0$. On the other hand, we have for every $n \geq 2$

$$0 < \varepsilon < \left(|f_n| - 4^n \sum_{i=1}^{n-1} |f_i|\right)^+ (T|x_n|) \leq g_n(T|x_n|) + 2^{-n} f(T|x_n|) \to 0,$$

which is impossible.

Now, for the "if " part, let $(x_n) \subset E$, $(f_n) \subset F'$ be respectively a disjoint weakly null and a disjoint weak* null sequences. We shall see by [4, Theorem 2.4 (3)] that $f_n(Tx_n) \to 0$. To this end, put $A = \{x_n : n \in \mathbb{N}\}$ and let $\varepsilon > 0$. By hypothesis there exists some $g \in (F')^+$ so that $(|f_n| - g)^+ (T|x_n|) \leq \varrho_{T,A}\left((|f_n| - g)^+\right) \leq \varepsilon$ holds for all n. As $|x_n| \xrightarrow{w} 0$ (Lemma 2.2), choose some natural number m such that $g(T|x_n|) \leq \varepsilon$ holds for every $n \geq m$. Thus, for every $n \geq m$ we get

$$\begin{aligned} |f_n(Tx_n)| &\leq |f_n|(T|x_n|) \\ &\leq (|f_n| - g)^+ (T|x_n|) + g(T|x_n|) \\ &\leq 2\varepsilon. \end{aligned}$$

This show that $f_n(Tx_n) \to 0$, and then T is a weak almost limited operator. □

Consequently, σ-Dedekind complete Banach lattices with the wDP* property enjoy the following lattice approximation property.

Corollary 2.6. *A σ-Dedekind complete Banach lattice E has the wDP* property if, and only if, each weak* null sequence $(f_n) \subset E'$ is approximately order bounded with respect to the lattice seminorm ϱ_A for every relatively weakly compact set $A \subset E$.*

The following main result gives some characterizations of positive weak almost limited operators (related to sequences with positive terms in statements (6)–(8)).

Theorem 2.7. *Let E and F be two Banach lattices such that F is σ-Dedekind complete. Then for a positive operator $T : E \to F$, the following assertions are equivalent:*

(1) *T is weak almost limited.*
(2) *$f_n(Tx_n) \to 0$ for every weakly null sequence $(x_n) \subset E^+$ and every disjoint weak* null sequence $(f_n) \subset F'$.*
(3) *$f_n(Tx_n) \to 0$ for every disjoint weakly null sequence $(x_n) \subset E^+$ and every disjoint weak* null sequence $(f_n) \subset F'$.*
(4) *$f_n(Tx_n) \to 0$ for every disjoint weakly null sequence $(x_n) \subset E^+$ and every disjoint weak* null sequence $(f_n) \subset (F')^+$.*
(5) *$f_n(Tx_n) \to 0$ for every disjoint weakly null sequence $(x_n) \subset E$ and every weak* null sequence $(f_n) \subset F'$.*

(6) $f_n(Tx_n) \to 0$ *for every weakly null sequence* $(x_n) \subset E^+$ *and every weak* null sequence* $(f_n) \subset F'$.
(7) $f_n(Tx_n) \to 0$ *for every weakly null sequence* $(x_n) \subset E$ *and every weak* null sequence* $(f_n) \subset (F')^+$.
(8) $f_n(Tx_n) \to 0$ *for every weakly null sequence* $(x_n) \subset E^+$ *and every weak* null sequence* $(f_n) \subset (F')^+$.

Proof. $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4)$ Obvious.

$(4) \Rightarrow (1)$ Follows from ([4], Theorem 2.4 (1 ⇔ 7)).

$(1) \Rightarrow (6)$ Let $(x_n) \subset E^+$, $(f_n) \subset F'$ be respectively a weak null and weak* null sequences, and let $\varepsilon > 0$. Put $A = \{x_n : n \in N\}$. From Theorem 2.5, pick some $g \in (F')^+$ so that $(|f_n|-g)^+(Tx_n) \leq \varrho_{T,A}\left((|f_n|-g)^+\right) \leq \varepsilon$ holds for all n, and choose some natural number m such that $g(Tx_n) < \varepsilon$ holds for every $n \geq m$. Now, for every $n \geq m$ we have

$$|f_n(Tx_n)| \leq |f_n|(Tx_n) \leq (|f_n|-g)^+(Tx_n) + g(Tx_n) \leq 2\varepsilon.$$

This shows that $f_n(Tx_n) \to 0$.

$(6) \Rightarrow (4)$ Obvious.

$(6) \Rightarrow (5)$ If $(x_n) \subset E$ is a disjoint weakly null sequence then by Lemma 2.2, we have $x_n^+ \xrightarrow{w} 0$ and $x_n^- \xrightarrow{w} 0$ and the result follows from the equality $f_n(Tx_n) = f_n(Tx_n^+) - f_n(Tx_n^-)$.

$(5) \Rightarrow (4)$ Obvious.

$(5) \Rightarrow (7)$ Let $(x_n) \subset E$, $(f_n) \subset (F')^+$ be respectively a weak null and weak* null sequences, and let $\varepsilon > 0$. We claim in this case that there exist $z \in E^+$ and a natural number k such that

$$f_n\left(T\left((|x_n|-z)^+\right)\right) < \varepsilon \tag{$*$}$$

holds for all $n > k$. To see this, assume by way of contradiction that $(*)$ is false. That is, for each $z \in E^+$ and each k we have $f_n\left(T\left((|x_n|-z)^+\right)\right) \geq \varepsilon$ for at least one $n > k$. An easy inductive argument shows that there exist a subsequence of (x_n) and a subsequence of (f_n) (which we still denote (x_n) and (f_n)) such that

$$f_n\left(T\left(|x_n| - 2^n \sum_{i=1}^{n-1} |x_i|\right)^+\right) \geq \varepsilon$$

holds for all $n \geq 2$. Let $x = \sum_{n=1}^{\infty} 2^{-n}|x_n|$ and $y_n = \left(|x_n| - 2^n \sum_{i=1}^{n-1}|x_i| - x\right)^+$. Clearly, $0 \leq y_n \leq |x_n|$ holds for every $n \geq 2$, and note that from Lemma 2.1 (y_n) is a disjoint sequence. Then by Lemma 2.2 we get $y_n \xrightarrow{w} 0$. Now, from our hypothesis we have $f_n(Ty_n) \to 0$. Or for every $n \geq 2$ we have

$$0 < \varepsilon \leq f_n\left(T\left(|x_n| - 2^n \sum_{i=1}^{n-1}|x_i|\right)^+\right) \leq f_n(Ty_n) + f_n(Tx) \to 0,$$

which is impossible. Therefore, $(*)$ is true.

Now, let $z \in E^+$ and let k be such that $(*)$ is valid, and choose $m > k$ such that $f_n(T(z)) < \varepsilon$ holds for every $n \geq m$. Thus, for every $n \geq m$ we have

$$|f_n(Tx_n)| \leq f_n(T|x_n|) \leq f_n(T(|x_n| - z)^+) + f_n(Tz) \leq 2\varepsilon.$$

This shows that $f_n(Tx_n) \to 0$.

(7) $\Rightarrow$ (4) Obvious.

(6) $\Rightarrow$ (8) $\Rightarrow$ (4) Obvious. □

From the statements (6) or (7) or (8) of Theorem 2.7, it follows easily the following corollaries.

Corollary 2.8. *Let E, F and G be a Banach lattices such that both F and G are σ-Dedekind complete. If for the scheme of positive operators $E \xrightarrow{T} F \xrightarrow{R} G$, T or R is weak almost limited then, so is the product RT. In particular if E is a σ-Dedekind complete Banach lattice, then the square of each positive weak almost limited operator $T : E \to E$ is likewise weak almost limited.*

Corollary 2.9. *If E and F are a σ-Dedekind complete Banach lattices such that E or F has the wDP* property, then each positive operator $T : E \to F$ is weak almost limited.*

The following corollary gives some new characterizations of the wDP* property of a σ-Dedekind complete Banach lattice, other than those established in [3, Theorem 3.2].

Corollary 2.10. *Let E be a σ-Dedekind complete Banach lattice. Then the following assertions are equivalent:*

(1) *E has the wDP* property.*
(2) *$f_n(x_n) \to 0$ for every disjoint weak null sequence $(x_n) \subset E$ and every weak* null sequence $(f_n) \subset E'$.*
(3) *$f_n(x_n) \to 0$ for every weakly null sequence $(x_n) \subset E^+$ and every weak* null sequence $(f_n) \subset E'$.*
(4) *$f_n(x_n) \to 0$ for every weakly null sequence $(x_n) \subset E$ and every weak* null sequence $(f_n) \subset (E')^+$.*
(5) *$f_n(x_n) \to 0$ for every weak null sequence $(x_n) \subset E^+$ and every weak* null sequence $(f_n) \subset (E')^+$.*

Corollary 2.11. *Let $T : E \to F$ be a positive operator from a Banach lattice E into a σ-Dedekind complete Banach lattice F. If the lattice operations of E are sequentially weakly continuous (resp. the lattice operations of F' are sequentially weak* continuous), then the following statements are equivalent:*

(1) *T is weak almost limited.*
(2) *$f_n(Tx_n) \to 0$ for every weakly null sequence $(x_n) \subset E$ and every weak* null sequence $(f_n) \subset F'$.*

Proof. (1) $\Rightarrow$ (2) Let $(x_n) \subset E$ and $(f_n) \subset F'$ be respectively weak null and weak* null sequences. We shall see that $f_n(Tx_n) \to 0$.

- If the lattice operations of E are sequentially weakly continuous, then the sequences (x_n^+) and (x_n^-) are both weak null. Thus, since T is weak almost limited, by Theorem 2.7(6) we have $f_n(Tx_n^+) \to 0$ and $f_n(Tx_n^-) \to 0$. Now, the result follows from the equality $f_n(Tx_n) = f_n(Tx_n^+) - f_n(Tx_n^-)$.
- If the lattice operations of F' are sequentially weak* continuous, then the sequences (f_n^+) and (f_n^-) are both weak* null. Thus, since T is weak almost limited, by Theorem 2.7(7) we have $f_n^+(Tx_n) \to 0$ and $f_n^-(Tx_n) \to 0$, and the result follows from the equality $f_n(Tx_n) = f_n^+(Tx_n) - f_n^-(Tx_n)$.

$(2) \Rightarrow (1)$ Obvious. $\square$

Note that a Banach lattice which has the wDP* property needs not to have the DP* one (eg $L^1[0,1]$, see [3, Proposition 3.3]). However, from the preceding theorem, another corollary can be derived easily.

Corollary 2.12. *Let E be a σ-Dedekind complete Banach lattice such that the lattice operations of E are sequentially weakly continuous, or the lattice operations of E' are sequentially weak* continuous. Then E has the wDP* property if and only if it has the DP* property.*

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