Arab Journal of
Mathematical Sciences

Arab J Math Sci 19(1) (2013), 105–107

CORRIGENDUM

Corrigendum to: Existence of solutions for multi point boundary value problems for fractional differential equations

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Available online 13 December 2012

Published online: Arab Journal of Mathematical Sciences 18 (2012) 165–175

We have made some mistakes in this paper. Here, we must substitute the main problem with

$$\begin{cases} D^{\alpha}_{0+}u(t) = f(t, u(t)), \quad t \in [0,1], \\ u(0) = u'(0) = 0, \quad u(1) - \sum_{i=1}^{m} a_i u(\xi_i) = \lambda \end{cases} \tag{1}$$

where D^{α}_{0+} is the Riemann–Liouville fractional derivative of order $2 < \alpha \leqslant 3$ and $m \geqslant 1$ is an integer, $\lambda \in (0, \infty)$ is a parameter, and a_i, ξ_i, f satisfying

(H1) $a_i > 0$ for $1 \leqslant i \leqslant m$, $0 < \xi_1 < \xi_2 < \cdots < \xi_n < 1$ and $\sum_{i=1}^{m} a_i \xi_i^{\alpha-1} < 1$;
(H2) $f : [0,1] \times [0,\infty) \to [0,\infty)$ is continuous.

Hence, we replace "$f(t, u(t), u'(t))$" by "$f(t, u(t))$" in this paper and correct Theorem 2, Example 3 and end of proof of Lemma 6, as follow:

Lemma 6. *$T : K \to K$ is a completely continuous operator.*

Proof. I correct the end of this Lemma as follow:

We have showed that T is a completely continuous operator. The operator T is completely continuous by an application of the Ascoli-Arzela theorem. □

DOI of original article: 10.1016/j.ajmsc.2012.01.002.
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[1] N.N. thanks Razi University for support.
Peer review under responsibility of King Saud University.

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http://dx.doi.org/10.1016/j.ajmsc.2012.11.001

We use the following notations:

$$M = \frac{\Gamma(\alpha)}{\Gamma(2\alpha)}\left(1 + \frac{\sum_{i=1}^{n} a_i}{\Gamma(\alpha)(1-\Delta)}\right)$$

$$R = \min_{\gamma \leqslant t \leqslant \delta}\left\{\int_{\gamma}^{\delta} G(t,s)ds + \frac{\sum_{i=1}^{n} a_i}{\Gamma(\alpha)(1-\Delta)}\int_{\gamma}^{\delta} G(\xi_i,s)ds\right\}$$

We are now ready to state our main results.

Theorem 2. *Suppose that there exist nonnegative numbers a,b,c such that* $0 < a < b < \sigma c$, *and f(t,u), satisfy the following conditions:*

(H3) $f(t,u) \leqslant \frac{c}{M}$, *for all* $(t,u) \in [0,1] \times [0,c]$;
(H4) $f(t,u) \leqslant \frac{a}{M}$, *for all* $(t,u) \in [0,1] \times [0,a]$;

(H5) $f(t,u) > \frac{b}{R}$, *for all* $(t,u) \in [\gamma,\delta] \times [b,\frac{b}{\sigma}]$. *In addition, suppose that* λ *satisfies*

$$0 < \lambda < \frac{c(1-\Delta)}{2}. \tag{2}$$

Then the problem (1) has at least three positive solutions u_1, u_2, u_3 such that $\|u_1\| < a, b < \alpha(u2(t))$ and $\|u_3\| > a$, with $\alpha(u_3(t)) < b$.

1. Application

Example 3. Consider the following fractional boundary value problem:

$$\begin{cases} D_{0+}^{\frac{5}{2}} u(t) = f(t,u(t)), & t \in [0,1] \\ u(0) = u'(0) = 0, & u(1) - \frac{1}{4}u\left(\frac{1}{3}\right) - \frac{3}{4}u\left(\frac{2}{3}\right) = \lambda \end{cases} \tag{3}$$

where

$$f(t,u) = \begin{cases} \frac{\sqrt{t}}{30} + \sin(\pi t) + u^6, & t \in [0,1], \ 0 \leqslant u < 2 \\ \frac{\sqrt{t}}{30} + \sin(\pi t) + 64 + \frac{15}{2}\sqrt{u-2}, & t \in [0,1], \ 2 \leqslant u < 18 \\ \frac{\sqrt{t}}{30} + \sin(\pi t) + 94 + \frac{15}{2}\sqrt{u-18}, & t \in [0,1], \ u \geqslant 18. \end{cases}$$

To show that the problem (3) has at least three positive solutions, we apply Theorem 2 with $\alpha = \frac{5}{2}$, $m = 2$, $a_1 = \frac{1}{4}$, $a_2 = \frac{3}{4}$, $\xi_1 = \frac{1}{3}$ and $\xi_2 = \frac{2}{3}$.

We choose $\gamma = \frac{1}{3}$ and $\delta = \frac{2}{3}$. Then, by direct calculations, we can obtain that

$$\Delta = 0.4564, \quad M = 0.264048, \quad R = 0.18297$$

By calculating, we can let $m_1 = \frac{\sqrt[3]{3-2\sqrt{2}}-2}{\sqrt[3]{3-2\sqrt{2}}-3}$ and $\sigma \simeq 0.01437$. If we take $a = 1$, $b = 2$ and $c = 100$, we finally obtain

$$f(t,u) \leqslant 104.3833 \leqslant \frac{c}{M} = 378.719, \quad \text{for all } 0 \leqslant t \leqslant 1,\ 0 \leqslant u \leqslant 100,$$

$$f(t,u) \leqslant 2.0333 \leqslant \frac{a}{M} = 3.677, \quad \text{for all } 0 \leqslant t \leqslant 1,\ 0 \leqslant u \leqslant 1,$$

$$f(t,u,v) \geqslant 64.5471 > \frac{b}{R} = 10.9307, \quad \text{for all } \frac{1}{3} \leqslant t \leqslant \frac{2}{3},\ 2 \leqslant u \leqslant 4.559.$$

Therefore, using Theorem 2 for $0 < \lambda \leqslant \frac{c(1-\Delta)}{2} = 27.18$, the problem (1) has at least three positive solutions u_i, $i = 1,2,3$, such that $\|u_1\| < 1,2 < \alpha(u2)$ and $\|u_3\| > 1$, with $\alpha(u_3) < 2$.