

CORRIGENDUM

**Corrigendum to: Existence of solutions for multi point
boundary value problems for fractional differential equations**

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We have made some mistakes in this paper. Here, we must substitute the main problem with

$$\begin{cases} D_{0+}^{\alpha} u(t) = f(t, u(t)), & t \in [0, 1], \\ u(0) = u'(0) = 0, & u(1) - \sum_{i=1}^m a_i u(\xi_i) = \lambda \end{cases} \quad (1)$$

where D_{0+}^{α} is the Riemann–Liouville fractional derivative of order $2 < \alpha \leq 3$ and $m \geq 1$ is an integer, $\lambda \in (0, \infty)$ is a parameter, and a_i, ξ_i, f satisfying

(H1) $a_i > 0$ for $1 \leq i \leq m$, $0 < \xi_1 < \xi_2 < \dots < \xi_m < 1$ and $\sum_{i=1}^m a_i \xi_i^{\alpha-1} < 1$;

(H2) $f: [0, 1] \times [0, \infty) \rightarrow [0, \infty)$ is continuous.

Hence, we replace “ $f(t, u(t), u'(t))$ ” by “ $f(t, u(t))$ ” in this paper and correct Theorem 2, Example 3 and end of proof of Lemma 6, as follow:

Lemma 6. $T: K \rightarrow K$ is a completely continuous operator.

Proof. I correct the end of this Lemma as follow:

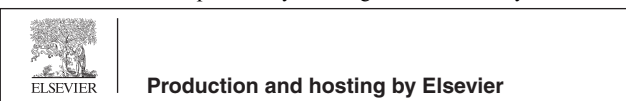
We have showed that T is a completely continuous operator. The operator T is completely continuous by an application of the Ascoli-Arzelà theorem. \square

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We use the following notations:

$$M = \frac{\Gamma(\alpha)}{\Gamma(2\alpha)} \left(1 + \frac{\sum_{i=1}^n a_i}{\Gamma(\alpha)(1-\Delta)} \right)$$

$$R = \min_{\gamma \leq t \leq \delta} \left\{ \int_{\gamma}^{\delta} G(t,s)ds + \frac{\sum_{i=1}^n a_i}{\Gamma(\alpha)(1-\Delta)} \int_{\gamma}^{\delta} G(\xi_i,s)ds \right\}$$

We are now ready to state our main results.

Theorem 2. *Suppose that there exist nonnegative numbers a, b, c such that $0 < a < b < \sigma c$, and $f(t, u)$, satisfy the following conditions:*

(H3) $f(t, u) \leq \frac{c}{M}$, for all $(t, u) \in [0, 1] \times [0, c]$;

(H4) $f(t, u) \leq \frac{a}{M}$, for all $(t, u) \in [0, 1] \times [0, a]$;

(H5) $f(t, u) > \frac{b}{R}$, for all $(t, u) \in [\gamma, \delta] \times [b, \frac{b}{\sigma}]$. In addition, suppose that λ satisfies

$$0 < \lambda < \frac{c(1-\Delta)}{2}. \tag{2}$$

Then the problem (1) has at least three positive solutions u_1, u_2, u_3 such that $\|u_1\| < a, b < \alpha(u_2(t))$ and $\|u_3\| > a$, with $\alpha(u_3(t)) < b$.

1. APPLICATION

Example 3. Consider the following fractional boundary value problem:

$$\begin{cases} D_{0+}^{\frac{5}{3}} u(t) = f(t, u(t)), & t \in [0, 1] \\ u(0) = u'(0) = 0, & u(1) - \frac{1}{4}u(\frac{1}{3}) - \frac{3}{4}u(\frac{2}{3}) = \lambda \end{cases} \tag{3}$$

where

$$f(t, u) = \begin{cases} \frac{\sqrt{t}}{30} + \sin(\pi t) + u^6, & t \in [0, 1], \quad 0 \leq u < 2 \\ \frac{\sqrt{t}}{30} + \sin(\pi t) + 64 + \frac{15}{2}\sqrt{u-2}, & t \in [0, 1], \quad 2 \leq u < 18 \\ \frac{\sqrt{t}}{30} + \sin(\pi t) + 94 + \frac{15}{2}\sqrt{u-18}, & t \in [0, 1], \quad u \geq 18. \end{cases}$$

To show that the problem (3) has at least three positive solutions, we apply Theorem 2 with $\alpha = \frac{5}{2}, m = 2, a_1 = \frac{1}{4}, a_2 = \frac{3}{4}, \xi_1 = \frac{1}{3}$ and $\xi_2 = \frac{2}{3}$.

We choose $\gamma = \frac{1}{3}$ and $\delta = \frac{2}{3}$. Then, by direct calculations, we can obtain that

$$\Delta = 0.4564, \quad M = 0.264048, \quad R = 0.18297$$

By calculating, we can let $m_1 = \frac{\sqrt[3]{3-2\sqrt{2}-2}}{\sqrt[3]{3-2\sqrt{2}-3}}$ and $\sigma = 0.01437$. If we take $a = 1, b = 2$ and $c = 100$, we finally obtain

$$f(t, u) \leq 104.3833 \leq \frac{c}{M} = 378.719, \quad \text{for all } 0 \leq t \leq 1, 0 \leq u \leq 100,$$

$$f(t, u) \leq 2.0333 \leq \frac{a}{M} = 3.677, \quad \text{for all } 0 \leq t \leq 1, 0 \leq u \leq 1,$$

$$f(t, u, v) \geq 64.5471 > \frac{b}{R} = 10.9307, \quad \text{for all } \frac{1}{3} \leq t \leq \frac{2}{3}, 2 \leq u \leq 4.559.$$

Therefore, using Theorem 2 for $0 < \lambda \leq \frac{c(1-\Delta)}{2} = 27.18$, the problem (1) has at least three positive solutions u_i , $i = 1, 2, 3$, such that $\|u_1\| < 1, 2 < \alpha(u_2)$ and $\|u_3\| > 1$, with $\alpha(u_3) < 2$.