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Book Review

The Fibonacci Resonance and Other New Golden Ratio Discoveries. Clive N. Menhinick. OnPerson International Limited, Poynton, Cheshire (2015). 618 pp.+xiv, ISBN 978-0-9932166-0-2

Fibonacci numbers form a popular sequence, even known to non-mathematicians. They show up in nature and in mathematical context, sometimes quite unexpectedly. This is partly due to the very simple recurrence relation $F_{n+1} = F_n + F_{n-1}$ with starting values $F_0 = 0, F_1 = 1$. The Lucas numbers form another interesting sequence defined by $L_{n+1} = L_n + L_{n-1}$ with starting values $L_0 = 2, L_1 = 1$. For the sake of simplicity, Fibonacci and Lucas numbers can be written explicitly using so called Binet's formulas

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right] \quad \text{and}$$

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n,$$

which can also be extended to negative integer values of n.

Among number theorists, the Pell and Pell-Lucas numbers are well known siblings of these, solving the recurrences $P_{n+1} = 2P_n + P_{n-1}$, with initial conditions $P_0 = 0, P_1 = 1$, respectively $Q_{n+1} = 2Q_n + Q_{n-1}$, with $Q_0 = 2, Q_1 = 2$. The last two sequences naturally arise in the study of the quadratic equation $x^2 - 2y^2 = (-1)^n$, whose solutions are $(Q_n/2, P_n)$, but also in many other situations.

This is a delightful book which should prove of great value, not only to the professional mathematician, but also to a great variety of other professionals who are interested in music, archaeology, architecture, art, quasicrystals, metamaterials, physics, history, biology and astronomy, to name but a few. The book is aimed at a large audience, hence the ideas and techniques used do not require any deep mathematical background. It contains a rich mixture of various fascinating topics, all connected to the Fibonacci numbers, and all pointing to the surprising significance of these numbers in nature, science, and art.

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The book is divided into six parts. In Parts I and II, the author takes the reader through some of the interesting occurrences of Fibonacci numbers and the Golden Ratio $\Phi = \frac{1+\sqrt{5}}{2}$, in nature, music, art, architecture, and Ori32 geometry. After having set this motivational stage, Part III is devoted to the Fibonacci resonance (Chapters 13–19). More precisely, the author presents standard inductive proofs to the "resonance formulae" for Fibonacci and Lucas numbers:

$$F_n = F_s\left(\frac{1+\sqrt{5}}{2}\right)^{n-s} + F_{n-s}\left(\frac{1-\sqrt{5}}{2}\right)^{s},$$

$$L_n = \sqrt{5}F_s\left(\frac{1+\sqrt{5}}{2}\right)^{n-s} + L_{n-s}\left(\frac{1-\sqrt{5}}{2}\right)^{s}, \quad n, s \in \mathbb{Z},$$

and gives the equivalent versions in terms of the Golden Ratio Φ. Some interesting consequences of the obtained Binet's type formulas are discussed. Part IV is entitled "To Pell and beyond" and it explores the generalized Fibonacci resonance formulae and some interpretations and applications. Part V, on "Φ in science", discusses the connection of the Golden Ratio Φ in quantum mechanics, molecular Φ, Penrose tiling, quasicrystals, Islamic tiling, superlattices and metamaterials, etc. The last part, "Appendices", supplies the content of the book with some mathematical notions and results: Ori32 trigonometry, Fibonacci hexads modulo 32, continued fractions, powers of Φ, the Binet's formula from a generating function, Mersenne primes, Hilbert's 10th problem.

The book ends with a Glossary, a list of the symbols used, a list of formulae, a list of figures, and an index. A very rich list of references containing 1004 titles is included. Although this is an extensive list, I have noticed that two basic references are missing. These are

1. T. Koshy, *Pell and Pell-Lucas Numbers with Applications*, Springer, 2014.
2. N.N. Vorobiev, *Fibonacci Numbers*, Birkhäuser, 2002.

The book is well written, well researched, well organized, and its coverage is truly extensive. It represents a comprehensive collection of results, theorems, illustrations in unexpected situations, and references regarding Fibonacci numbers and their applications to date. The timing and scope of the book make it a rather fitting tribute to the enduring impact of the original Fibonacci's "Liber Abaci". The style of presentation is lively and precise. I recommend the present book without reservation to professional mathematicians, for many interesting non-mathematical examples and applications, that can be used in teaching a general course on algebra, number theory, and discrete mathematics, to professional scientists and engineers, to students, and to the general amateurs and enthusiasts alike.

Overall, I enjoyed the book and highly recommend it.

Dorin Andrica