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An Osgood condition for a semilinear reaction–diffusion equation with time-dependent generator

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Abstract. In this note we give a sufficient condition for blow up of positive mild solutions to an initial value problem for a non-autonomous equation with fractional diffusion.

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1. INTRODUCTION

In this paper we study blow up of positive mild solutions of

$$\begin{aligned} &\frac{\partial}{\partial t}u\left(t,x\right)=k\left(t\right)\Delta_{\alpha}u\left(t,x\right)+h\left(t\right)R(u\left(t,x\right)), \quad t>0,\ x\in\mathbb{R}^{d}, \\ &\quad u\left(0,x\right)=\varphi\left(x\right), \quad x\in\mathbb{R}^{d}, \end{aligned} \tag{1}$$

where $\Delta_\alpha = -\left(-\Delta/2\right)^{\alpha/2}$, $0<\alpha\leq 2$, is the α-Laplacian, $\varphi:\mathbb{R}^d\to\mathbb{R}$ is a function non negative, bounded, continuous and $k,h,R:[0,\infty)\to[0,\infty)$ are continuous functions.

If there exists a solution u of (1) defined in $[0,\infty)\times\mathbb{R}^d$, we say that u is a (classical) global solution, on the other hand if there exists a number $t_e<\infty$ such that u is unbounded in $[0,t]\times\mathbb{R}^d$, for each $t>t_e$, we say that u blows up in finite time.

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It is well known [4] that the associated integral system of (1) is

$$u(t,x) = \int_{\mathbb{R}^d} p\left(K(t), y-x\right)\varphi(y)dy + \int_0^t \int_{\mathbb{R}^d} p\left(K(s,t), y-x\right) h\left(s\right) R(u\left(s,y\right))dyds. \tag{2}$$

Here $p\left(t,x\right)$ denotes the fundamental solution of $\frac{\partial}{\partial t} - \Delta_\alpha$, this function is symmetric in the spatial component (we state more properties of $p\left(t,x\right)$ in Lemma 7). We also define

$$K\left(s,t\right) = \int_s^t k\left(r\right)dr, \quad 0 \le s \le t,$$

where $K(t) := K(0,t)$.

We say that u is a mild solution of (1) if u is a solution of (2). The main result is:

Theorem 1. *Assume the previous hypotheses on φ, k, h and R. Also suppose:*

$(H1)$ *R is an increasing and convex function in $[0,\infty)$,*

$(H2)$ $\int_1^\infty \frac{ds}{R(s)} < \infty$,

$(H3)$ $\lim_{t\to\infty} \frac{k(t)}{K(t)h(t)} = L \in [0,\infty)$,

$(H4)$ $\int_1^\infty h(s)ds = \infty$,

$(H5)$ $\lim_{t\to\infty} \frac{\exp\left(c\int_1^t h(s)ds\right)}{(K(1,t))^{d/\alpha}} = \infty$, *for each $c > 0$.*

Let $J : [0,\infty) \to [0,\infty)$ be a continuous function satisfying

$(H6)$ *J is submultiplicative, i.e., there exists a constant $\tilde{a} > 0$ such that $J(x)J(y) \ge \tilde{a}J(xy)$, for each $x, y > 0$,*

$(H7)$ $\lim_{x\downarrow 0} \frac{R(x)}{J(x)} = \tilde{L} \in (0,\infty]$,

$(H8)$ $\int_1^\infty \frac{h(s)(2K(s))^{d/\alpha}}{J((2K(s))^{d/\alpha})}ds = \infty$ *and* $\int_1^\infty \frac{ds}{J(s)} < \infty$.

Then all non-trivial positive solutions of (2) blow up in finite time.

The importance of the study of equations like (1) is well known in applied mathematics. For example, they arise in fields like molecular biology, hydrodynamics and statistical physics [7]. Also, notice that generators of the form $g_i\left(t\right)\Delta_{\alpha_i}$ arises in models of anomalous growth of certain fractal interfaces [3].

There are many related works (see for instance [1,5,6] and the references cited therein) and some of them are contained in the following:

Example 2. Assume that

$$\begin{aligned} k(t) &= t^{\rho-1}, \\ h(t) &= t^{\sigma-1}, \\ R(x) &= x^{1+\beta}, \\ J(x) &= x^p + x^q, \end{aligned}$$

where ρ, σ, β, p and q are positive constants. If

$$1+\beta \le q \le p \quad \text{and} \quad \sigma \ge (q-1)\frac{\alpha\rho}{d}$$

then each non trivial solution of (2) blows up in finite time. If we take $p = q = 1 + \beta$, then we have Theorem 2 in [4] as particular case. We also have, as particular case, the blow up result in [8] when

$$1 \leq q \leq p \quad \text{and} \quad p \leq 1 + \frac{\alpha}{d}$$

and $\rho = 1, \sigma = 1, q = p$. In this case R is a convex function satisfying the hypotheses in Theorem 1.

Let us present others interesting examples.

Example 3. Let us take

$$\begin{aligned} k(t) &= 1, \\ h(t) &= t^{1+d/\alpha}, \\ R(x) &= e^x, \\ J(x) &= (1+x)^2. \end{aligned}$$

In this case there is blows up in finite time of the mild solution of the corresponding integral equation. However, observe that hypothesis (F.2) in [8] is not satisfied since

$$\lim_{x \downarrow 0} \frac{e^x}{x^{1+\gamma}} \notin (0, \infty)$$

for each $\gamma \in \mathbb{R} \backslash \{-1\}$. Therefore the criterion in [8] can not be applied.

In the above examples the submultiplicative function J is a power, but this is not necessary. Several examples of submultiplicative functions are presented in [2], one of them is the function $\log(e + x)$. Observe that $a_1 x^{b_1} (\log(x + e))^{c_1} + a_2 x^{b_2} (\log(x + e))^{c_2}$, is also submultiplicative if $a_i, b_i, c_i \geq 0$, $i = 1, 2$.

Example 4. Now take

$$\begin{aligned} k(t) &= e^{-t}, \\ h(t) &= 1, \\ R(x) &= (x+e)(\log(x+e))^{1+\beta}, \\ J(x) &= R(x), \end{aligned}$$

where $\beta > 0$. This example is quite informative because, contrary to the above example, now the contribution of the reaction component and the diffusion component is small, but large enough $(R(x) > x)$ so that there is blows up in finite time of the mild solution of the corresponding integral equation.

The paper is organized as follows. In Section 1 we prove the existence of local solutions for Eq. (2). In Section 2 we give some preliminary results and we reduce the study of explosion to the case of the time component, finally in Section 3 we prove the main result.

2. LOCAL EXISTENCE

The existence of local solutions to (2) follows form the Banach contraction principle, as we shall see.

Let us introduce some normed linear spaces. By $L^{\infty}\left(\mathbb{R}^d\right)$ we denote the space of all real-valued functions essentially bounded defined on $\mathbb{R}^d$. Let $\tau > 0$ be a real number that we will fix later. Define

$$E_{\tau} = \left\{ u : [0,\tau] \to L^{\infty}\left(\mathbb{R}^d\right),\ \|\!|u|\!\| < \infty \right\},$$

where

$$\|\!|u|\!\| = \sup \left\{ \|u\left(t\right)\|_{\infty} : 0 \leq t \leq \tau \right\}.$$

Then E_{τ} is a Banach space and the sets, $r > 0$,

$$P_{\tau} = \left\{ u \in E_{\tau} : u \geq 0 \right\}, \qquad B_{\tau,r} = \left\{ u \in E_{\tau} : \|\!|u|\!\| \leq r \right\},$$

are closed subspaces of E_{τ}.

Theorem 5. *Let $\varphi : \mathbb{R}^d \to \mathbb{R}$ be a bounded non negative function, $k, h : [0,\infty) \to [0,\infty)$ are continuous functions and $R : [0,\infty) \to [0,\infty)$ is a convex function. There exists a $\tau > 0$ such that the integral equation (2) has a local solution in $B_{\tau,r} \cap P_{\tau}$.*

Proof. Define the operator $\Psi : B_{\tau,r} \cap P_{\tau} \to E_{\tau}$, by

$$\begin{aligned}\Psi\left(u\right)\left(t,x\right) = &\int_{\mathbb{R}^d} p\left(K\left(t\right), y - x\right)\varphi\left(y\right)dy \\ &+ \int_0^t \int_{\mathbb{R}^d} p\left(K\left(s,t\right), y - x\right)h\left(s\right)R(u\left(s,y\right))dyds.\end{aligned}$$

Since $\varphi \geq 0$, $u \geq 0$, it is clear that $\Psi\left(u\right) \geq 0$, then $\Psi\left(u\right) \in P_{\tau}$. For each $t \in [0,\tau]$, $x \in \mathbb{R}^d$ and $u \in B_{\tau,r}$,

$$\begin{aligned}\Psi\left(u\right)\left(t,x\right) \leq &\|\varphi\|_{\infty} \int_{\mathbb{R}^d} p\left(K\left(t\right), y - x\right)dy \\ &+ \int_0^t \int_{\mathbb{R}^d} p\left(K\left(s,t\right), y - x\right)h\left(s\right)R(\|u\left(s\right)\|_{\infty})dyds \\ \leq &\|\varphi\|_{\infty} + \int_0^t \int_{\mathbb{R}^d} p\left(K\left(s,t\right), y - x\right)h\left(s\right)R(r)dyds \\ = &\|\varphi\|_{\infty} + R(r)\int_0^{\tau} h\left(s\right)ds,\end{aligned}$$

then

$$\|\!|\Psi\left(u\right)|\!\| \leq \|\varphi\|_{\infty} + R(r)\int_0^{\tau} h\left(s\right)ds.$$

Let us take $r = 1 + \|\varphi\|_\infty$. Since

$$\lim_{t\to 0}\int_0^t h(s)\,ds = 0, \tag{3}$$

we can choose $\tau > 0$ small enough such that

$$R(r)\int_0^\tau h(s)\,ds < 1.$$

Then $\Psi(u) \in B_{\tau,r} \cap P_\tau$, therefore $\Psi(B_{\tau,r} \cap P_\tau) \subset B_{\tau,r} \cap P_\tau$.

Now let us see that Ψ is a contraction. Let $u, \tilde{u} \in B_{\tau,r} \cap P_\tau$,

$$\begin{aligned}
&|\Psi(u)(t,x) - \Psi(\tilde{u})(t,x)| \\
&\quad = \left|\int_0^t \int_{\mathbb{R}^d} p(K(s,t), y-x)\, h(s)\,[R(u(s,y)) - R(\tilde{u}(s,y))]\,dyds\right| \\
&\quad \le \sup_{t\in[0,\tau]} \int_0^t \int_{\mathbb{R}^d} p(K(s,t), y-x)\, h(s)\,|R(u(s,y)) - R(\tilde{u}(s,y))|dyds.
\end{aligned}$$

Since R is convex on $[0,r]$, we have for each $0 \le x < y \le r$,

$$\frac{R(y)-R(x)}{y-x} \le D_l R(r),$$

where $D_l R(r)$ is the left-hand derivative of R at r (it always exist for convex functions, see Theorem 14.5 in [9]). From this we can deduce

$$\begin{aligned}
|R(u(s,y)) - R(\tilde{u}(s,y))| &\le D_l R(r)\,|u(s,x) - \tilde{u}(s,x)| \\
&\le D_l R(r)\|u(s) - \tilde{u}(s)\|_\infty,
\end{aligned}$$

this turns out

$$\begin{aligned}
|||\,\Psi(u) - \Psi(\tilde{u})\,||| &\le \sup_{t\in[0,\tau]} \int_0^t h(s)\, D_l R(r)\,\|u(s) - \tilde{u}(s)\|_\infty\, ds \\
&\le \left\{ D_l R(r) \int_0^\tau h(s)\,ds \right\} |||u - \tilde{u}|||.
\end{aligned}$$

Using again (3) we can choose $\tau > 0$ small enough such that Ψ is a contraction. Therefore Ψ has a unique fixed point, the local solution to the integral equation (2). ■

3. Preliminary results

We begin recalling a basic integral inequality:

Theorem 6 (*Jensen's Inequality*)**.** *Let $(X, \mathcal{A}, \mu)$ be a finite measure space and f a real-valued integrable function. If φ is a convex function on an open interval I in $\mathbb{R}$ and if $f(X) \subset I$, then*

$$\varphi\left(\frac{1}{\mu(X)}\int_X f\,d\mu\right) \le \frac{1}{\mu(X)}\int_X (\varphi\circ f)\,d\mu.$$

Proof. See Theorem 14.16 in [9]. ■

Also we state some properties of α-stable densities:

Lemma 7. *For any $s,t>0$ and any $x,y\in\mathbb{R}^d$, we have*
(i) $p\,(ts,x)=t^{-d/\alpha}p\left(s,t^{-1/\alpha}x\right)$.
(ii) $p\,(t,x)\geq\left(\frac{s}{t}\right)^{d/\alpha}p\,(s,x)$, *for* $t\geq s$.

Proof. The proof can be seen in Section 2 of [8]. ■

The first step is to make a shift in the time for a proper instant, this instant is given by the following result.

Lemma 8. *Let u be a positive solution of (2), then*

$$u\,(t_0,x)\geq c(t_0)p\,(\gamma,x)\,,\quad\forall x\in\mathbb{R}^d,\tag{4}$$

where t_0, γ and $c(t_0)$ are positive constants.

Proof. See Lemma 1 in [4]. ■

Using the semigroup property and the previous time t_0 (see (4)) we have

$$\begin{aligned}
u(t+t_0,x) &= \int_{\mathbb{R}^d}p\,(K\,(t_0,t+t_0)\,,y-x)\,u(t_0,y)dy\\
&\quad+\int_0^t\int_{\mathbb{R}^d}p\,(K\,(s+t_0,t+t_0)\,,y-x)\,h\,(s+t_0)\,R(u\,(s+t_0,y))dyds\\
&\geq c(t_0)\int_{\mathbb{R}^d}p\,(K\,(t_0,t+t_0)\,,y-x)\,p\,(\gamma,y)\,dy\\
&\quad+\int_0^t\int_{\mathbb{R}^d}p\,(K\,(s+t_0,t+t_0)\,,y-x)\,h\,(s+t_0)\,R(u\,(s+t_0,y))dyds\\
&= c(t_0)p\,(K\,(t_0,t+t_0)+\gamma,x)\\
&\quad+\int_0^t\int_{\mathbb{R}^d}h\,(s+t_0)\,p\,(K\,(s+t_0,t+t_0)\,,y-x)\,R(u\,(s+t_0,y))dyds.
\end{aligned}$$

In order to eliminate the contribution of the spatial component, we multiply both sides of the previous inequality by $p\,(K\,(t+t_0)\,,x)$ and then we integrate with respect to x, afterwards we use Fubini's theorem and the semigroup property to get

$$\begin{aligned}
\bar{u}\,(t) &\geq c(t_0)\int_{\mathbb{R}^d}p(K\,(t+t_0)\,,x)p\,(K\,(t_0,t+t_0)+\gamma,x)\,dx\\
&\quad+\int_0^t h\,(s+t_0)\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}p(K\,(t+t_0)\,,x)p\,(K\,(s+t_0,t+t_0)\,,y-x)\\
&\quad\times R(u\,(s+t_0,y))dxdyds\\
&= c(t_0)p\,(K\,(t+t_0)+K\,(t_0,t+t_0)+\gamma,0)\\
&\quad+\int_0^t h\,(s+t_0)\int_{\mathbb{R}^d}p\,(K\,(t+t_0)+K\,(s+t_0,t+t_0)\,,y)\\
&\quad\times R(u\,(s+t_0,y))dyds,
\end{aligned}\tag{5}$$

where

$$\bar{u}(t) = \int_{\mathbb{R}^d} p(K(t+t_0), x)\, u(t+t_0, x)dx.$$

The desired condition, on blows up of (2), is a consequence of the following fact.

Lemma 9. *If $\overline{u}$ blows up in finite time, then u also does.*

Proof. See Lemma 2 in [4]. ■

4. Proof of Theorem 1

From (5) and using (ii) of Lemma 7 we get the underestimation

$$\begin{aligned}\bar{u}(t+t_0) \geq{}& c(t_0)p(K(t+t_0) + K(t_0, t+t_0) + \gamma, 0) \\ &+ \int_0^t \int_{\mathbb{R}^d} \left(\frac{K(s+t_0)}{K(t+t_0) + K(s+t_0, t+t_0)} \right)^{d/\alpha} p(K(s+t_0), y) \\ &\times h(s+t_0) R(u(s+t_0, y)) dyds\end{aligned}$$

and Jensen's inequality yields

$$\begin{aligned}\bar{u}(t+t_0) \geq{}& c(t_0)p(K(t+t_0) + K(t_0, t+t_0) + \gamma, 0) \\ &+ \int_0^t h(s+t_0) \left(\frac{K(s+t_0)}{K(t+t_0) + K(s+t_0, t+t_0)} \right)^{d/\alpha} R(\bar{u}(s+t_0))\, ds.\end{aligned}$$

Applying now the property (i) of Lemma 7 we obtain

$$\begin{aligned}\bar{u}(t+t_0) \geq{}& c(t_0)(K(t+t_0) + K(t_0, t+t_0) + \gamma)^{-d/\alpha} p(1, 0) \\ &+ \int_0^t h(s+t_0) \left(\frac{K(s+t_0)}{K(t+t_0) + K(s+t_0, t+t_0)} \right)^{d/\alpha} R(\bar{u}(s+t_0))\, ds \\ \geq{}& c(t_0)p(1, 0)(2K(t+t_0) + \gamma)^{-d/\alpha} \\ &+ \int_0^t h(s+t_0) \left(\frac{K(s+t_0)}{2K(t+t_0)} \right)^{d/\alpha} R(\bar{u}(s+t_0))\, ds.\end{aligned}$$

Let

$$v(t) = (2K(t+t_0))^{d/\alpha} \bar{u}(t+t_0), \quad t \geq 0,$$

then

$$\begin{aligned}v(t+t_0) \geq{}& c(t_0)p(1, 0)(1 + (2K(t_0))^{-1}\gamma)^{-d/\alpha} \\ &+ \frac{1}{2^{d/\alpha}} \int_0^t h(s+t_0)(2K(s+t_0))^{d/\alpha} R\left(\frac{v(s+t_0)}{(2K(s+t_0))^{d/\alpha}} \right) ds.\end{aligned}$$

Let us consider the integral equation

$$\bar{v}(t+t_0) = c + c\int_0^t h(s+t_0)(2K(s+t_0))^{d/\alpha} R\left(\frac{\bar{v}(s+t_0)}{(2K(s+t_0))^{d/\alpha}} \right) ds, \tag{6}$$

where c is a positive constant. Set

$$w(t+t_0) = (2K\,(t+t_0))^{-d/\alpha}\bar{v}(t+t_0), \quad t \geq 0.$$

Observe that, by (6),

$$\begin{aligned}\frac{d}{dt}w(t+t_0) &= ch(t+t_0)R\left(\frac{\bar{v}\,(t+t_0)}{(2K\,(t+t_0))^{d/\alpha}}\right)\\ &\qquad -\frac{d}{\alpha}2k\,(t+t_0)\,(2K\,(t+t_0))^{-\frac{d}{\alpha}-1}\bar{v}(t+t_0)\\ &= ch(t+t_0)R\,(w(t+t_0)) - \frac{dk\,(t+t_0)}{\alpha K\,(t+t_0)}w(t+t_0). \end{aligned} \tag{7}$$

We are going to prove that w blows up in finite time, but first we will see that w is not bounded.

Lemma 10. *We have* $\lim_{t\to\infty} w(t) = \infty$.

Proof of Lemma 10. Using the Hypothesis (H7) we can find $a_1, \delta > 0$, such that

$$\frac{R(x)}{J(x)} > a_1, \quad \forall x \in (0, \delta].$$

By Lemma 1 in [8] there exists a constant $M > 0$ large enough such that

$$\frac{R(x)}{x} > 1, \quad \forall x \in [M, \infty).$$

On the other hand, the continuity of $x^{-1}R(x)$ in $[\delta, M]$, implies that there exists $a_2 > 0$ for which

$$\frac{R(x)}{x} \geq a_2, \quad \forall x \in [\delta, M].$$

Choosing $a = \min\{a_1, a_2, 1\}$ we have

$$\max\left\{\frac{R(x)}{x}, \frac{R(x)}{J(x)}\right\} \geq a, \quad \forall x > 0.$$

The submultiplicative property of J (Hypothesis (H6)) yields

$$\begin{aligned}&(2K\,(s+t_0))^{d/\alpha}R\left(\frac{\bar{v}\,(s+t_0)}{(2K\,(s+t_0))^{d/\alpha}}\right)\\ &\quad\geq a\min\left\{\bar{v}\,(s+t_0)\,, \tilde{a}(2K\,(s+t_0))^{d/\alpha}\frac{J(\bar{v}\,(s+t_0))}{J((2K\,(s+t_0))^{d/\alpha})}\right\}.\end{aligned}$$

Now let us consider the equation

$$z(t) = c + c\int_0^t h(s+t_0)a\min\left\{z\,(s)\,, \tilde{a}\frac{(2K\,(s+t_0))^{d/\alpha}}{J((2K\,(s+t_0))^{d/\alpha})}J(z\,(s))\right\}ds.$$

By the comparison theorem we can see that

$$z(t) \geq \min\{z_1(t), z_2(t)\}, \quad t \geq 0,$$

where

$$z_1(t) = c + ca \int_0^t h(s+t_0) z_1(s)\, ds,$$
$$z_2(t) = c + ca\tilde{a} \int_0^t h(s+t_0) \frac{(2K(s+t_0))^{d/\alpha}}{J((2K(s+t_0))^{d/\alpha})} J(z_2(s)) ds.$$

The Hypothesis (H8) implies that z_2 blows up in finite time, therefore $z(t) \geq z_1(t)$ for t large enough. Then the result follows from Hypothesis (H5). ■

It is clear that $\bar{v}$ blows up in finite time if and only if w also does. Let us suppose that $w(t) < \infty$ for all $t \geq t_0$, then the Hypotheses (H1)–(H2) and the previous result implies (see Lemma 1 in [8])

$$\lim_{t\to\infty} \frac{w(t)}{R(w(t))} = 0,$$

then (H3) turns out

$$\lim_{t\to\infty} \frac{dk(t)\, w(t)}{\alpha K(t)\, h(t) R(w(t))} = 0.$$

Therefore, there exists a $t_1 > 0$ such that

$$\frac{dk(t)\, w(t)}{\alpha K(t)\, h(t) R(w(t))} < \frac{c}{2}, \quad \forall t \geq t_1 + t_0,$$

then (7) can be subestimated as

$$\frac{d}{dt} w(t+t_0) \geq \frac{c}{2} h(t+t_0) R(w(t+t_0)), \quad \forall t \geq t_1.$$

Let $\bar{w}$ be the solution to

$$\frac{d}{dt}\bar{w}(t) = \frac{c}{2} h(t) R(\bar{w}(t)), \quad t > t_0 + t_1,$$
$$\bar{w}(t_0+t_1) = c.$$

This implies

$$\int_{t_0+t_1}^t \frac{\bar{w}'(s)}{R(\bar{w}(s))} ds = \frac{c}{2} \int_{t_0+t_1}^t h(s) ds.$$

Doing the change of variable $z = \bar{w}(s)$ we have

$$\int_{\bar{w}(t_0+t_1)}^{\bar{w}(t)} \frac{ds}{R(s)} = \frac{c}{2} \int_{t_0+t_1}^t h(s) ds, \quad \forall t > t_0 + t_1.$$

The continuity of R and the non-negativity property of h imply that

$$\infty > \int_{\bar{w}(t_0+t_1)}^{\infty} \frac{ds}{R(s)} \geq \frac{c}{2} \int_{t_0+t_1}^{\infty} h(s) ds.$$

This contradicts the assumption H4. Therefore there exists a $t_e > t_0$ such that $\lim_{t\uparrow t_e} w(t) = \infty$. ■

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