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SEMITOPOLOGICAL SPACES

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ABSTRACT. Levine (1963) introduced the notions of semi-open set and semi-continuity in topological spaces and obtained a number of their properties. Biswas (1969) defined semi-open functions and investigated several properties of such functions. Also, Noiri (1973) investigated some properties of semi-continuty and semi-open functions. Latif (1998) introduced the concept of semitopology and explored some of its properties. In this paper we give a characterization of S-semi- continuous functions, S-irresolute functions, separation axioms, S-semi-closed graphs and strongly S-semi-closed graphs. It is shown that many results in previous papers can be considered as special cases of our results.

1. INTRODUCTION

Let X, Y, and Z be topological spaces on which no separation axioms are assumed unless explicity stated. Let S be a subset of X. The closure (resp. interior, boundary) of S will be denoted by $CI(S)$ (resp. $Int(S)$, $Bd(S)$). A subset S of X is called semi-open (Levine 1963) if $S \subseteq CI[Int(S)]$, the complement of a semi-open set is called semi-closed. The family of all semi-open sets in X will be denoted by $SO(X)$. The semi-interior of a set A is the largest semi-open set contained in A. A function $f : X \longrightarrow Y$ is called semi-continuous (Levine 1963) if $f^{-1}(V) \in SO(X)$ for each open set V of Y, f is irresolute (Crossley and Hildebrand 1971) if $f^{-1}(V) \in SO(X)$ for each $V \in SO(Y)$. A function $f : X \longrightarrow Y$ is called semi-open (Biswas 1969) if $f(U) \in SO(Y)$ for each open set U of X, and it is called semi-closed (Noiri 1973) if the image of each closed set in X is semi-closed in Y.

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2. SEMITOPOLOGIES

The following definitions are taken from our earlier papers of 1998.

Definition 1.1 Let (X,τ) be a topological space. Let $\gamma \subseteq SO(X,\tau)$. Then we define the semi-intersection of members of γ to be the semi-interior of $\cap\gamma$, denoted by $\cap^*\gamma$. If $A_1, A_2, ..., A_n \in SO(X,\tau)$, we will write $\cap^*\{A_1, A_2, ...A_n\} = A_1 \cap^* A_2 \cap^* ... \cap^* A_n$.

Definition 1.2 Let (X,τ) be a topological space. Let $\tau^* \subseteq SO(X,\tau)$. We say that τ^* is a semitopology on X if the following axioms are satisfied:
(S_1) $X, \phi \in \tau^*$,
(S_2) For every $A, B \in \tau^*, A \cap^* B \in \tau^*$,
(S_3) For each $\gamma \subseteq \tau^*, \cup\gamma \in \tau^*$.
We will call the ordered pair (X,τ^*) a semitopological space.

Definition 1.3 Let (X,τ) be a topological space and τ^* be a semitopology on X. A semi-open set A in X will be called S-semi-open in X if and only if $A \in \tau^*$.

Definition 1.4 Given a semi-topological space (X,τ^*), a semi-open subset E of X is said to be S-semi-closed in X when its complement $X - E$ in X is S-semi-open in X.

Various notations like the interior, closure, exterior and the derived set operators as well as set properties like dense, nowhere dense-in itself, compact etc. can be defined in a semitopological space in analogy with topological spaces. Many results of topological spaces remain valid in semitopological spaces, whereas some become false. The S-semi-derived set (resp. S-semi-closure, S-semi-interior, S-semi-boundary) of a subset S of a space X will be denoted by $sDer(S)$(resp. $sCl(S), sInt(S), sBd(S)$).

In what follows, we list the main properties of such operations which give the deviations between these operations and that in topological spaces.

Theorem 1.1 *Let (X,τ) be a topological space and let $\tau^* \subseteq SO(X,\tau)$ such that τ^* is a semitopology on X. Let A and B be subsets of X. Then*

(i) $sDer(A) \cup sDer(B) \subseteq sDer(A \cup B)$;
(ii) $sCI(A) \cup sCI(B) \subseteq sCI(A \cup B)$;
(iii) $sInt(A \cap B) \subseteq sInt(A) \cap sInt(B)$.

It can be easily shown, by examples, that the inequalities in (i), (ii) and (iii) cannot be replaced, in general, by equalities as in the case of topological spaces.

Definition 2.2 Let (X, τ) be a topological space and $\tau^* \subseteq SO(X, \tau)$ such that τ^* is a semitopology on X. We call τ^* a semitopology associated with τ if $\tau \subseteq \tau^*$.

Example. Let (X, τ) be any topological space. Then $\tau^* = SO(X, \tau)$ is a semitopology associated with τ.

Remark. The intersection of two S-semi-open sets need not be S-semi-open. But if $U \in \tau$ and $V \in \tau^*$, then $U \cap V \in \tau^*$.

2. S-SEMI-CONTINUOUS FUNCTIONS

Definition 2.1 Let (X, τ_1) (Y, τ_2) be topological spaces and τ_1^* be an associated semitopology with τ_1. We say that a function $f : X \to Y$ is an S-semi-continuous function if the inverse image of each open set in Y is S-semi-open in X.

Theorem 2.1 *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then the following are equivalent:*
(i) f is S-semi-continuous;
(ii) The inverse image of each closed set in Y is S-semi-closed in X;
(iii) $sCI[f^{-1}(V)] \subseteq f^{-1}[Cl(V)]$, for every $V \subseteq Y$;
(iv) $f[sCI(U)] \subseteq Cl[f(U)]$, for every $U \subseteq X$;
(v) For any point $x \in X$ and any open set V of Y containing $f(x)$, there exists $U \in \tau_1^*$ such that $x \in U$ and $f(U) \subseteq V$;
(vi) $sBd[f^{-1}(V)] \subseteq f^{-1}[Bd(V)]$, for every $V \subseteq Y$;
(vii) $f(sCl(U)] \subseteq Cl[f(U)]$, for every $U \subseteq X$;
(viii) $f^{-1}(Int(B)] \subseteq sInt[f^{-1}(B)]$, for every $B \subseteq Y$;

Proof. (i) $\Rightarrow$(ii). Let $F \subseteq Y$ be closed. Since f is S-semi-continuous, $f^{-1}(Y - F) = X - f^{-1}(F)$ is S-semi-open. Therefore, $f^{-1}(F)$ is S-semi-closed in X.

(ii) $\Rightarrow$ (iii). Since $Cl(V)$ is closed for every $V \subseteq Y$, then $f^{-1}[Cl(V)]$ is S-semi-closed. Therefore $f^{-1}[Cl(V)] = sCl[f^{-1}(Cl(V))] \supseteq sC(l[f - 1(V)]$.

(iii) $\Rightarrow$ (iv). Let $U \subseteq X$ and $f(U) = V$. Then $f^{-1}[Cl(V)] \supseteq sCl[f^{-1}(V)]$. Thus $f^{-1}[Cl(f(U))] \supseteq sCl[f^{-1}(f(U))] \supseteq sCl(U)$ and $Cl[f(U)] \supseteq f[sCl(U)]$.

(iv) $\Rightarrow$ (ii). Let $W \subseteq Y$ be a closed set, $U = f^{-1}(W)$, then $f[sCl(U)] \subseteq Cl[f(U)] = Cl[f(f^{-1}(W))] \subseteq Cl(W) = W$. $sCl(U) \subseteq f^{-1}[f(sCl(U))] \subseteq f^{-1}(W) = U, sCl(U) \subseteq U$. So U is S-semi-closed.

(ii) $\Rightarrow$ (i). Let $V \subseteq Y$ be an open set. Then $Y - V$ is closed. Therefore $f^{-1}(Y - V) = X - f^{-1}(V)$ is S-semi-closed in X and hence $f^{-1}(V)$ is S-semi-open in X.

(i) $\Rightarrow$ (v). Let $f : X \to Y$ be S-semi-continuous. For any $x \in X$ and any open set V of Y containing $f(x), U = f^{-1}(V) \in \tau_1^*$ and $f(U) = f[f^{-1}(V)] \subseteq V$.

(v) $\Rightarrow$ (i). Let $V \in \tau_2$. We prove that $f^{-1}(V) \in \tau_1^*$. Let $x \in f^{-1}(V)$. Then $f(x) \in (V)$ and there exists $U \in \tau_1^*$ such that $x \in U$ and $f(x) \in f(u) \subseteq V$. Hence $x \in U \subseteq f^{-1}[f(U)] \subseteq f^{-1}(V)$ and $f^{-1}(V)$ is S-semi-open neighborhood of each of its points. Therefore $f^{-1}(V) \in \tau_1^*$.

(i) $\Rightarrow$ (vi). Since $f^{-1}[Bd(V)] = f^{-1}[Cl(V) - Int(V)]$. For each $V \subseteq Y, f^{-1}[Bd(V)] = f^{-1}[Cl(V)] - f^{-1}[Int(V)] \supseteq sCl[f^{-1}(V)] - sInt[f^{-1}(Int(V))] \supseteq sCl[f^{-1}(V)] - sInt[f^{-1}(V)] = sBd[f^{-1}(V)]$.

(vi) $\Rightarrow$ (i). Let $U \subseteq Y$ be an open set, $V = Y - U$. Since V is closed, $sBd[f^{-1}(V)] \subseteq f^{-1}[Bd(V)] \subseteq f^{-1}[Cl(V)] = f^{-1}(V)$. Thus $f^{-1}(V)$ is S-semi-closed and f is S-semi-continuous.

(i) $\Rightarrow$ (vii). It is obvious, since f is S-semi-continuous and by (iv) $f[sCl(U)] \subseteq Cl[f(U)]$ for each $U \subseteq X$. So $f[sDer[(U)] \subseteq Cl[f(U)]$.

(vii) $\Rightarrow$ (i). Let $U \subseteq Y$ be an open set, $V = Y - U$ and $f^{-1}(V) = W$. Then $f[sDer(W)] \subseteq Cl[f(W)]$. Thus $f[sDer(f^{-1}(V))] \subseteq Cl[f(f^{-1}(V))] \subseteq Cl(V) = V$. Then $sDer[f^{-1}(V)] \subseteq f^{-1}(V)$ and $f^{-1}(V)$ is S-semi-closed.

Therefore, f is S-semi-continuous.

(i) $\Rightarrow$ (viii). Let $B \subseteq Y$. Then $f^{-1}[Int(B)]$ is S-semi-open in X. But $f^{-1}[Int(B)] = sInt[f^{-1}(Int(B))] \subseteq sInt[f^{-1}(B)]$. Therefore, $f^{-1}[Int(B)] \subseteq sInt[f^{-1}(B)]$.

(viii) $\Rightarrow$ (i). Let $V \subseteq Y$ be an open set. Then $f^{-1}(V) = f^{-1}[Int(V)] \subseteq sInt[f^{-1}(V)]$. Therefore $f^{-1}(V)$ is S-semi-open. Hence f is S-semi-continuous.

Remarks

(i) Every continuous function is S-semi-continuous.
(ii) If a function $f : (X, \tau_1) \to (Y, \tau_2)$ is S-semi-continuous and a function $g : (Y, \tau_2) \to (Z, \tau_3)$ is S-semi-continuous, then $gof : (X, \tau_1) \to (Z, \tau_3)$ may not be S-semi-continuous.
(iii) If a function $f : (X, \tau_1) \to (Y, \tau_2)$ is S-semi-continuos and $g : (Y, \tau_2) \to (Z, \tau_3)$ is continuous, then (gof) is S-semi-continuous.
(iv) Let (X, τ_1) and (Y, τ_2) be topological spaces, τ_1^* and τ_2^* be two associated semitopologies with τ_1 and τ_2, respectively. If $f : X \to Y$ is a function, and one of the following properties holds
(a) $f^{-1}[sInt(B)] \subseteq Int[f^{-1}(B)]$, for each $B \subseteq Y$,
(b) $Cl[f^{-1}(B)] \subseteq f^{-1}[sCl(B)]$, for each $B \subseteq Y$,
(c) $f[Cl(A)] \subseteq sCl[f(A)]$, for each $A \subseteq X$,
then f is continuous.

Important Remark. Many properties of semi-continuity (Levine 1963) can be easily deduced from the previous results of S-semi-continuity by setting $\tau^* = SO(X, \tau)$.

3. S-IRRESOLUTE FUNCTIONS

Definition 3.1 Let $(X, \tau_1), (Y, \tau_2)$ be topological spaces and τ_1^* and τ_2^* be two associated semitopologies with τ_1, τ_2, respectively. Let $f : X \to Y$ be a function. Then we say that f is S-irresolute if the inverse image of each S-semi-open set is S-semi-open.

From the above definition, one can draw the following diagram:

Continuity $\Rightarrow$ S-semi-continuity$\Leftarrow$ S-irresolute
It is easy to give examples to show that the concepts of continuity and S-irresolute are independent.

Theorem 3.1 *Let $f : X \rightarrow Y$ be an injective S-irresolute function. Then $In[f(A)] \subseteq sInt[f(A)] \subseteq f[sInt(A)]$.*

Theorem 3.2 *Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions. If f is S-irresolute and g is S-semi-continuous, then $gof : X \rightarrow Z$ is S-irresolute.*

Theorem 3.3 *Let $f : X \rightarrow Y$ be a function. Then the following are equivalent:*
(i) f is S-irresolute.
(ii) The inverse image of each S-semi-closed set is S-semi-closed.
(iii) For each $A \subseteq X, f[sCl(A)] \subseteq sCl[f(A)] \subseteq Cl[f(A)]$.
(iv) For each $B \subseteq Y, sCl[f^{-1}(B)] \subseteq f^{-1}[sCl[B)] \subseteq f^{-1}[Cl(B)]$.
(v) For each $x \in X$ and each S-semi-open set $V \subseteq Y$ containing $f(x)$, there exists an S-semi-open set $U \subseteq X$ containing x such that $f(U) \subseteq V$.
(vi) For each $B \subseteq Y, sBd[f^{-1}(B)] \subseteq f^{-1}[sBd(B)] \subseteq f^{-1}[Bd(B)]$.
(vii) For each $A \subseteq X, f[sDer(A)] \subseteq sCl[f(A)] \subseteq Cl[f(A)]$.
(viii) For each $B \subseteq Y, f^{-1}[Int(B)] \subseteq f^{-1}[sInt(B)] \subseteq sInt[f^{-1}(B)]$.

Proof. Similar to that of Theorem 2.1.

Remarks
(i) Many properties of irresolute (Crossley and Hildebrand 1971) functions are special cases of previous results of S-irresolute functions by setting $\tau_1^* = SO(X)$ and $\tau_2^* = SO(Y)$.
(ii) One may also define S-semi-open, S-semi-closed, S-pre-semi-open and S-pre-semi-closed sets as well as S-semi-homeomorphic functions.

4. SEPARATION AXIOMS IN SEMITOPOLOGICAL SPACES

Definition 4.1. Let (X, τ) be a topological space and τ^* be a semitopology associated with τ. Then the semitopological space (X, τ^*) is called
(i) S-semi-T_0, if for every two points of X, there exists an S-semi-open neighborhood of one of them to which the other does not belong.
(ii) S-semi-T_1, if for every two distinct points x and y in X, there exist two

S-semi-open sets U and V such that $x \in U, y \notin U$ and $y \in V, x \notin V$.
(iii) S-semi-T_2, if for every two distinct points x and y in X, there exist two disjoint S-semi-open sets U and V such that $x \in U$ and $y \in V$.
(iv) S-semi-T_2', if for every two distinct points x and y in X, there exist two S-semi-open sets U and V such that $x \in U, y \in V$ and $Cl(U \cap Cl(V) = \Phi$.
From the above definitions, one can draw the following diagram:

$$\begin{array}{ccccccc} T_2' & \Rightarrow & T_2 & \Rightarrow & T_1 & \Rightarrow & T_0 \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ S\text{-}semi\text{-}T_2' & \Rightarrow & S\text{-}semi\text{-}T_2 & \Rightarrow & S\text{-}semi\text{-}T_1 & \Rightarrow & S\text{-}semi\text{-}T_0 \end{array}$$

It is easy to give examples to show that the inverse directions are not true in general.

Theorem 4.1 *Let (X, τ) be a topological space, and τ^* a semitopology on X associated with τ. Suppose the following properties hold:*
(i) If $x, y \in X$ such that $x \neq y$, then $SCl(\{x\}) \neq sCl(\{y\})$.
(ii) If $x \in X$, then $sDer(\{x\})$ is the union of S-semi-closed sets.
(iii) If $A \subseteq X$, then $A = \cap\{N : N \neq y$ is an S-semi-open-neighborhood of $A\}$.
(iv) If $x \in X$, then $\{x\}$ is S-semi-closed.
(v) If $x \neq y$, then there exists an S-semi-closed neighborhood of x to which y does not belong..
(vi) If $x \in X$, then $\{x\} = \cap\{N : N$ is an S-semi-closed-neighborhood of $x\}$.
Then
(a) S-semi-$T_0 \Longleftrightarrow$ (i) $\Longleftrightarrow$ (ii).
(b) S-semi-$T_1 \Longleftrightarrow$ (iii) $\Longleftrightarrow$ (iv).
(c) S-semi-$T_2 \Longleftrightarrow$ (v) $\Longleftrightarrow$ (vi).

Remark. Many properties of semi-T_i spaces (Maheshewari and Prasad 1975) can be easily obtained from the properties of S-semi-T_i spaces by setting $\tau^* = SO(X, \tau), i = 0, 1, 2$.

Theorem 4.2 *If X is an S-semi-T_2'-space and $x \neq y$, then there exist two S-semi-open neighborhoods N_x of x and N_y of y such that $N_x' \cap N_y' = \Phi$ (N_x' is the set of all limit points of N_x).*
The converse of Theorem 4.2, is not true in general. But it is true in the case of $\tau^* = SO(X)$.

Theorem 4.3 *The property of being an S-semi-T_i-space is S-semi-topological property,* $i = 0, 1, 2$.

Definition 4.2 A function $f : X \to Y$ is said to be S-pre-semi-open if and only if the image of each S-semi-open set is an S-semi-open set.

Theorem 4.4 *The property of being S-semi-T_2 is preserved under S-pre-semi-open bijection.*

Remark. Since the intersection of an open set and an S-semi-open set is always S-semi-open, it follows that every open (S-semi-open) subspace of S-semi-T_0 (resp. S-semi-T_1, S-semi-T_2, S-semi-T_2') space is S-semi-T_0 (resp. S-semi-T_1, S-semi-T_2, S-semi-T_2') space.

5. S-SEMICLOSED GRAPHS AND STRONGLY S-SEMI-CLOSED GRAPHS

Definition 5.1 A subset A of the product space $X \times Y$ is S-semi-closed in $X \times Y$ if for each $(x, y) \in X \times Y - A$, there exist two S-semi-open neighborhoods U and V of x and y, respectively, such that $(U \times V) \cap A = \Phi$.
A function $f : X \to Y$ has an S-semi-closed graph, if the graph $G(f) = \{(x, f(x)) : x \in X\}$ is S-semi-closed in $X \times Y$.

Lemma 5.1 *A function $f : X \to Y$ has an S-semi-closed graph if and only if for each $x \in X, y \in Y$ such that $y \neq f(x)$, there exist two S-semi-open sets U and V containing x and y, respectively such that $f(U) \cap V = \Phi$.*

Proof. **Necessity.** Since f has an S-semi-closed graph, then for each $x \in X$ and $y \in Y$ such that $y \neq f(x)$, there exist two S-semi-open sets U and V containing x and y, respectively such that $(U \times V) \cap G(f) = \Phi$. This implies that for every $x \in U$ and $y \in V, f(x) \neq y$. So $f(U) \cap V = \Phi$.
Sufficiency. Let $(x, y) \notin G(f)$. Then there exist two S-semi-open sets U and V containing x and y, respectively such that $f(U) \cap V = \Phi$. This implies that for each $x \in U$ and $y \in V$, we have $f(x) \neq y$. So $(U \times V) \cap G(f) = \Phi$. Hence f has an S-semi-closed graph.

Theorem 5.1 *If $f : X \to Y$ is S-irresolute and Y is an S-semi-T_2-space, then f has an S-semi-closed graph.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$, and since Y is S-semi-T_2, there exist two S-semi-open sets U and V asuch that $f(x) \in U$, $y \in V$ and $U \cap V = \Phi$. Since f is S-irresolute, there exists an S-semi-open neighborhood W of x such that $f(W) \subseteq U$. Hence $f(W) \cap V = \Phi$. This implies that $(W \times V) \cap G(f) = \Phi$. Hence f has an S-semi-closed graph.

Theorem 5.2 *If $f : X \to Y$ is S-irresolute injection with an S-semi-closed graph, then X is S-semi-T_2.*

Proof. Let $x_1, x_2 \in X$ such that $x_1 \neq x_2$. Then $f(x_1) \neq f(x_2)$. This implies that $(x_1, f(x_2)) \in (X \times Y) - G(f)$. Since f has an S-semi-closed graph, there exist two S-semi-open neighborhoods U and V of x_1 and $f(x_2)$, respectively such that $(U \times V) \cap G(f) = \Phi$. This gives $f(U) \cap V = \Phi$. Since f is S-irresolute, there exists an S-semi-open set W containing x_2 such that $f(W) \subseteq V$. Hence $f(W) f(U) = \Phi$. Therefore $W \cap U = \Phi$ and X is an S-semi-T_2-space.

Definition 5.2 A function $f : X \to Y$ has a strongly S-semi-closed graph if for each $(x, y) \notin G(f)$, there exist two S-semi-open sets U and V containing x and y, respectively, such that $(U \times sCl(V)) \cap G(f) = \Phi$.

Lemma 5.2 *A function $f : X \to Y$ has a strongly S-semi-closed graph if for each $(x, y) \notin G(f)$, there exist two S-semi-open neighborhoods U and V of x and y, respectively such that $f(U) \cap sCl(V) = \Phi$.*

Definition 5.3 Let (X, τ) be a topological space. Then a subset S of X is called an α-set if $S \subseteq Int[Cl(Int(S))]$.

Definition 5.4 A function $f : X \to Y$ has an α-closed graph if and only for each $x \in X, y \in Y$ such that $y \neq f(x)$, there exist two α-sets U and V in X and Y, respectively such that $x \in U$ and $y \in V$ such that $f(U) \cap Cl(V) = \Phi$.

Lemma 5.3 *A function $f : X \to Y$ has an α-closed graph if and only if for each $x \in X$ and $y \in Y$ such that $y \neq f(x)$, there exist an α-set U in X and an open set V in Y such that $x \in U$ and $y \in V$ satisfying $f(U) \cap Cl(V) = \Phi$.*

Proof. **Necessity.** Since f has an α-closed graph, then for each $x \in X$ and $y \in Y$ such that $y \neq f(x)$, there exist two α-sets U and V such that $x \in U, y \in V$ and $f(U) \cap Cl(V) = \Phi$. Let $W = Int[Cl(Int(V))]$. Then

$y \in W$ and W is open. Since V is an α-set, so clearly $Cl(W) \subseteq Cl(V)$. Thus $f(U) \cap Cl(W) = \Phi$.
Sufficiency. It follows from the fact that every open set is an α-set.

Corollary 5.1 *Suppose that $f : X \to Y$ has an α-closed graph. Then f has a strongly S-semi-closed graph.*

Proof. Let x $\in X$ and $y \in Y$ such that $f(x) \neq y$. Since f has an α-closed graph, then by Lemma 5.3, there exist an α-set U and an open set V such that $x \in U, y \in V$ and $f(U) \cap Cl(V) = \Phi$. Since every open set is S-semi-open, $sCL(V) \subseteq Cl(V)$, then $f(U) \cap sCl(V) = \Phi$. So f has a strongly S-semi-closed graph.

The connection between closed graphs, strongly closed graphs (Long and Herrington 1975), -closed graphs and the new types of graphs can be shown from the following diagram:

strongly closed graphs		$\Rightarrow$		closed graphs
$\Downarrow$				$\Downarrow$
α-closed graphs	$\Rightarrow$	strongly S-semi-closed graphs	$\Rightarrow$	S-semi-closed graphs

Theorem 5.3 *If $f : X \to Y$ is S-irresolute and Y is an S-semi-T_2'-space, then f has a strongly S-semi-closed graph.*

Proof. Let $(x, y) \notin G(f)$. Then $y \neq f(x)$. Since Y is S-semi-T_2', there exist two S-semi-open neighborhoods U and V of y and $f(x)$, respectively, such that $Cl(U) \cap Cl(V) = \Phi$. Since f is S-irresolute, there exists an S-semi-open neighborhood W of x such that $f(W) \subseteq U \subseteq Cl(U)$. So $f(W) \cap Cl(V) = \Phi$. But $sCl(V) \subseteq Cl(V)$, then $f(W) \cap sCl(V) = \Phi$. Hence, f has a strongly S-semi-closed graph.

Theorem 5.4 *If $f : X \to Y$ is S-pre-semi-open and has an S-semi-closed graph, then f has a strongly S-semi-closed graph.*

Proof. Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Since f has an S-semi-closed graph, there exist two S-semi-open neighborhoods U and V of x and y, respectively, such that $f(U) \cap V = \Phi$. Since f is S-pre-semi-open, then $f(U) \cap sCl(V) = \Phi$. Therefore f has a strongly S-semi-closed graph.

Theorem 5.5 *Let $f : X \to Y$ be a surjective function with a strongly S-semi-closed graph. Then Y is S-semi-T_2-space.*

Proof. Let y_1 and y_2 be distinct points in Y. Then there exists an $x_1 \in X$ such that $f(x_1) = y_1$. Thus $(x_1, y_2) \notin G(f)$. Since f has a strongly S-semi-closed graph, then there exist two S-semi-open neighborhoods U and V of x_1 and y_2 respectively such that $f(U) \cap sCl(V) = \Phi$. Consequently $y_1 \notin sCl(V)$. So X is S-semi-T_2-space.

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