Arab J. Math. Sc.
Volume 10, Number 2, December 2004
pp.59-69

ABOUT GEVREY-L^2-ESTIMATES OF PSEUDO-DIFFERENTIAL OPERATORS ASSOCIATED TO THE GEVREY SYMBOLS

MOHAMMED HAZI

ABSTRACT. Many authors have been interested in the generalisation of the fundamental theorems on the L^2 and H^s continuity of pseudo-differential operators in C^∞ and analytic classes. We can quote, Beals [1], Calderon, and Vaillancourt [4], Coifman, and Meyer [5], Hwang [8], and Rodino [11]. In [2], Boulkhemair gave a survey of these results and improved several of them. To our knowledge, the Gevrey regularity of these operators is relatively less explored. Nevertheless, Boutet de Monvel, and Krée [3], Hazi [7], and Matsuzawa [10] have investigated such problem. The starting-point of the present study is a result mentionned by Taylor, [12] chapter XIII. More precisely, we reconsider the corresponding Gevrey class analogue and see if it remains true. The answer is in positive. Namely, in our proof we revisit the arguments of Calderon and Vaillancourt [4] and others, and provide a precise value of the constant in the estimates, with an easy application to the Gevrey context.

1. PRELIMINARIES

1.1. NOTATIONS. In the sequel, we will use the following conventions:

- $\mathbb{R}^n$ is the n-dimensional vector space in which every point x is defined by its n coordinates $x_1, x_2, \cdots, x_n$.
- Ω denotes, unless expressed otherwise, an open set of $\mathbb{R}^n$.

AMS subject classification: 35S05,47B35,47G10.
Key words: Gevrey, kernel, L^2-continuity, pseudo-diffrential operators, symbol.

- $x+y$ is the point with coordinates $x_1+y_1, x_2+y, \cdots, x_n+y_n$.
- dx refers to the element of hypercube $dx_1 dx_1 \cdots dx_n$.
- The order of a system of integers $p=\{p_1,p_2,\cdots,p_n\}$ is $|p|=p_1+p_2+\cdots+p_n$.
- $D^\alpha = i^{-|\alpha|}\dfrac{\partial^{\alpha_1}}{\partial x_1^{\alpha_1}}\dfrac{\partial^{\alpha_2}}{\partial x_2^{\alpha_2}}\cdots\dfrac{\partial^{\alpha_n}}{\partial x_n^{\alpha_n}}$.
- $\Delta_x = \dfrac{\partial^2}{\partial x_1^2}+\dfrac{\partial^2}{\partial x_2^2}+\cdots+\dfrac{\partial^2}{\partial x_n^2}$.
- $p! = p_1!p_2!\cdots p_n!$.
- $\widehat{u}$ stands for the Fourier transform of u.
- A^* is the adjoint of the operator A.
- $\mathcal{E}(\mathbb{R}^n)$ is the space of indefinitely differentiable functions on $\mathbb{R}^n$.
- $\mathcal{D}(\Omega)$ is the space of indefinitely differentiable functions on $\mathbb{R}^n$, with compact support in Ω.

1.2. CLASS OF GEVREY-TYPE FONCTIONS.

Definition 1.1 ([6], [7]). Let s be a real number greater than or equal to 1. A real function f in $C^\infty(\Omega)$ is said to be of Gevrey class with order s if, for any compact subset $K\subset\Omega$, there exists a constant $C>0$ such that

$$\|D^\alpha f\| \le C^{|\alpha|+1}\left(|\alpha|!\right)^s, \qquad \forall\alpha\in\mathbb{N}^n.$$

1.3. GEVREY SYMBOLS OF CLASS s OF TYPE (m,ρ,δ).

Definition 1.2. Let $m\in\mathbb{R}$ and ρ,δ two real numbers such that $0\le\delta<\rho\le 1$. We say that a real function $a=a(x,\xi)$ in $C^\infty(\Omega\times\mathbb{R}^n)$ is a Gevrey symbol with order s of type (m,ρ,δ) on Ω if, for any compact subset $K\subset\Omega$, there exist positive constants C_0, C_1, B such that

$$(1.1)\qquad \underset{(x,\xi)\in(K\times\mathbb{R}^n)}{Sup}\left|D_\xi^\alpha D_x^\beta a(x,\xi)\right| \le C_0 C_1^{|\alpha+\beta|}\left(|\alpha|!\right)^s\left(|\beta|!\right)^s\left(1+|\xi|^2\right)^{\frac{1}{2}(m-\rho|\alpha|+\delta|\beta|)},$$

for any $\xi\in R^n$ and any $\alpha,\beta\in N^n$.

The vector space of such symbols, sometimes called usual (or classical symbols), is refered to as ${}_{\rho,\delta}S^m_{(G,s)}(\Omega \times \mathbb{R}^n)$.

In the sequel, we are concerned with the class of symbols $(m, 1, 1)$. It is to point out here that the function $a(x, \xi) = a$ is taken of Gevrey class with order s in x and ξ, whereas, often in the literature (see in particular [3]), it is taken of Gevrey class with order s in x and analytic $(s = 1)$ in ξ (which amounts to take $s = 1$ in the factor $(|\alpha|!)^s$).

The following theorem gives the asymptotic extension of a symbol.

Theorem 1.1. *Let a_j be a symbol of ${}_{\rho,\delta}S^{m_j}_{(G,s)}(\Omega \times \mathbb{R}^n)$, where $(m_j)_j$ is a real sequence decreasing to $-\infty$. Then, there exists a symbol a of ${}_{\rho,\delta}S^{m_0}_{(G,s)}(\Omega \times \mathbb{R}^n)$ such that, for any $N > 0$, we have*

$$a - \sum_{0}^{N-1} a_j \in {}_{\rho,\delta}S^{m_N}_{(G,s)}(\Omega \times \mathbb{R}^n).$$

This fact is also expressed by

$$a \sim \sum_{0}^{\infty} a_j.$$

1.4. PSEUDO-DIFFERENTIAL OPERATORS OF CLASS s.

A pseudo-differential operator of class s, $A = a(x, D)$, associated to a symbol a of the space ${}_{\rho,\delta}S^m_{(G,s)}(\Omega \times \mathbb{R}^n)$ is defined, relatively to the standard quantization, by the formula

$$a(x, D)u(x) = (2\pi)^{-n} \int e^{ix\xi} a(x, \xi) \widehat{u}(\xi) d\xi, \qquad u \in D(R^n).$$

We write $A = opa$ and say that A belongs to $Op_{\rho,\delta}S^m_{(G,s)}(\Omega \times \mathbb{R}^n)$.

The distribution-kernel T of $a(x, D)$ is defined by

$$T(x, y) = (2\pi)^{-n} \int_{\mathbb{R}^n} e^{i(x-y)\xi} a(x, \xi) d\xi.$$

2. CHRONOLOGICAL RECALL OF SOME RESULTS

Among the considerable results devoted to the L^2-continuity of pseudo-differential operators in the case of $C^\infty-$quantizations, (see [2] in particular), we recall

Theorem 2.1. *$A = a(x, D)$ sends $L^2(\mathbb{R}^n)$ into itself continuously whenever*

$$\left\| D_x^\beta D_\xi^\alpha a(x,\xi) \right\| \leq C_{\alpha\beta} \tag{2.1}$$

for all multi-indices α, β such that $|\alpha|$, $|\beta| \leq 3n + 4$ ($C_{\alpha\beta}$ being a positive constant).

In addition, if we set

$$\|A\|_0 = \underset{|\alpha|,|\beta| \leq 3n+4}{Sup} C_{\alpha\beta}$$

where $C_{\alpha\beta}$ are given by (1.1), then

$$\|a(x,D)u\|_{L^2(\mathbb{R}^n)} \leq C \|A\|_0 \|u\|_{L^2(\mathbb{R}^n)}$$

where C is a positive constant depending only on n.

Theorem 2.2. *$a(x, D)$ defines a bounded operator on $L^2(\mathbb{R}^n)$ whenever*

$$D_x^\beta D_\xi^\alpha a(x,\xi) \in L^\infty(\mathbb{R}^n \times \mathbb{R}^n)$$

for all multi-indices α, β such that $|\alpha|, |\beta| \leq \left[\frac{n}{2}\right]$ or $\alpha, \beta \in \{0,1\}^n$. ($[u]$ denotes the integer part of the real u).

In 1972, Calderon and Vaillancourt [4] proved the following result :

Theorem 2.3. *$a(x, D)$ is bounded on $L^2(\mathbb{R}^n)$ if there exists δ such that $0 \leq \delta < 1$ and*

$$\left| D_x^\beta D_\xi^\alpha a(x,\xi) \right| \leq C\,|\xi|^{\delta(|\beta|-|\alpha|)}$$

for $|\alpha| \leq n + 2\left[\frac{n}{2}\right]$ and $|\beta| \leq 2N$, $\quad N \geq \frac{5n}{4(1-\delta)}$.

In 1978, Coifman and Meyer [5] improved this result:

Theorem 2.4. *an opa is bounded on $L^2(\mathbb{R}^n)$ if there exists δ such that $0 \leq \delta < 1$ and*

$$\left| D_x^\beta D_\xi^\alpha a(x,\xi) \right| \leq C\,|\xi|^{\delta(|\beta|-|\alpha|)}; \qquad for \qquad |\alpha|, |\beta| \leq \left[\frac{n}{2}\right] + 1.$$

In 1987, Hwang [8] proved the following:

Theorem 2.5. *an opa is bounded on $L^2(\mathbb{R}^n)$ if there exists δ such that $0 \leq \delta < 1$ and*

$$\left|D_x^\beta D_\xi^\alpha a(x,\xi)\right| \leq C\,|\xi|^{\delta(|\beta|-|\alpha|)}$$

for $\alpha_j = 0$ or 1 and $\beta_j = 0$ or 1 if $n=1$ and $\beta_j = 0, 1$ or 2 in general.

3. MAIN RESULT

In what follows, we prove

Theorem 3.1. *Assume $a(x,\xi) \in {}_{\delta,\delta}S^m_{(G,s)}(\mathbb{R}^n \times \mathbb{R}^n)$, $m \leq 0$, $0 \leq \delta < 1$, and*

$$\underset{x\in\mathbb{R}^n}{Sup}\left|D_x^\beta D_\xi^\alpha a(x,\xi)\right| \leq C_0 C_1^{|\alpha+\beta|}\left(|\alpha|!\right)^s\left(|\beta|!\right)^s\left(1+|\xi|^2\right)^{m-\delta(|\alpha|-|\beta|)} \tag{3.1}$$

for any $\xi \in \mathbb{R}^n$ and $|\alpha|$, $|\beta| \leq 3n+4 = N$. Then, the operator $A = a(x,D)$ acts continuously from $L^2(\mathbb{R}^n)$ into itself. Moreover, if

$$\|A\|_\delta = \underset{|\alpha|,|\beta|\leq N}{Sup}\; C_0 C_1^{|\alpha+\beta|}\left(|\alpha|!\right)^s\left(|\beta|!\right)^s,$$

we get

$$\|a(x,D)u\|_{L^2(\mathbb{R}^n)} \leq C\,\|A\|_\delta\,\|u\|_{L^2(\mathbb{R}^n)}$$

where C is a positive constant depending only on δ.

3.1. **Proof of Theorem 3.1.** It is sufficient to prove this theorem for $a \in {}_{\delta,\delta}S^0_{G^s}(\mathbb{R}^n \times \mathbb{R}^n)$. We make use of two results. The first one is the famous Cotlar-Stein lemma on sums of almost orthogonal operators, whereas the second one is the well-known Schur's test for the boundedness of integral operators.

First, we recall the definition of almost orthogonal operators:

Definition 3.1. We call a familly of continuous operators

$$\{A_i : i \in \mathbb{Z}\}$$

almost orthogonal if they satisfy the following conditions:

$$\|A_i^* A_j\| \leq a(i,j), \qquad \|A_i A_j^*\| \leq b(i,j),$$

where $a(i,j)$ and $b(i,j)$ are non negative symmetric functions on $\mathbb{Z}\times\mathbb{Z}$ which satisfy

$$\|a\|_{\infty,1/2}^{1/2} = \sup_{i\in\mathbb{Z}}\sum_{j\in\mathbb{Z}} a^{1/2}(i,j) < \infty, \qquad \|b\|_{\infty,1/2}^{1/2} = \sup_{i\in\mathbb{Z}}\sum_{j\in\mathbb{Z}} b^{1/2}(i,j) < \infty.$$

Now, we give Cotlar-Stein's lemma:

Lemma 3.1. *Let A_1, A_2,$\cdots$, A_N be bounded operators from a Hilbert space H_1 into another H_2 such that*

$$(3.2)\qquad \begin{cases} \sum_k \sqrt{\left\|A_j^*A_k\right\|} \le M, \\ \\ \sum_k \sqrt{\|A_jA_k^*\|} \le M, \end{cases}$$

where M is a positive constant. Then it folows that

$$\left\|\sum_k A_j\right\| \le M.$$

Proof. If $A=\sum A_j$, we have

$$\|A\|^2 = \|A^*A\|\,,$$

and more generaly, by the spectral mapping theorem, we have

$$\|A\|^{2m} = \|(A^*A)^m\|\,.$$

We expand into a sum and use the fact that

$$\left\|A_{j1}^*A_{j2}\cdots A_{j2m-1}^*A_{j2m}\right\| \le$$
$$\min\left\{\|A_{j1}^*A_{j2}\|\cdots\left\|A_{j2m-1}^*A_{j2m}\right\|, \|A_{j1}^*\|\,\|A_{j2}A_{j3}^*\|\cdots\left\|A_{j2m-2}A_{j2m-1}^*\right\|\,\|A_{j2m}\|\right\}$$

□

Taking the geometric mean of the two estimates and noting that $\|A_j\|\le M$, by hypothesis, we obtain

$$\|A\|^{2m} \le M\sum\sqrt{\left\|A_{j1}^*A_{j2}\right\|}\sqrt{\left\|A_{j2}A_{j3}^*\right\|}\cdots\sqrt{\left\|A_{j2m-1}^*A_{j2m}\right\|}$$

The sum is taken over $j_1, j_2, \cdots, j_{2m}$. If we use (2.2) to estimate successivley the sum over $j_{2m}, j_2, \cdots, j_2$, then only the sum over j_1 is left over and we see that

$$\|A\|^{2m} = NM^{2m}.$$

Taking $2m$-th roots and, letting m go to ∞, we get $\|A\| \leq M$, as expected.

We also give the following variant of Schur's test for boundedness of integral operators:

Lemma 3.2. *Let X be a measurable space. Assume $K(x,y)$ to be a kernel-distribution satisfying*

$$\begin{aligned} \int_X |K(x,y)|\, dy &\leq C_0, \\ \int_X |K(x,y)|\, dx &\leq C_1, \end{aligned}$$

where C_0 and C_1 are two positive constants. Then $Pu(x) = \int K(x,y)u(y)dy$ defines a continuous operator on $L^2(X)$; moreover

$$\|P\| \leq \sqrt{C_0 C_1}.$$

Proof. Using Cauchy-Schwarz inequality as well as elementary estimates, we infer that

$$\begin{aligned} |\langle Pu, v\rangle| &\leq \int |K(x,y)|\,|u(x)|\,|v(y)|\, dxdy \\ &= \int \left(\sqrt{|K(x,y)|}\,|u(x)|\right)\left(\sqrt{|K(x,y)|}\,|v(y)|\right) dxdy \\ &\leq \sqrt{\int |K(x,y)|\,|u(x)|^2\, dydx}\sqrt{\int |K(x,y)|\,|v(y)|^2\, dxdy} \\ &\leq \sqrt{C_0}\,\|u\|_{L^2}\sqrt{C_1}\,\|v\|_{L^2}, \end{aligned}$$

and the claim follows. □

Now, let us turn back to the proof of our main theorem (Theorem 3.1).

We shall put the operator A under the form of a sum of quasi-orthogonal operators $A = \sum A_j$.

To this end, we choose a partition of the unity φ_j on $[0, \infty[$, $j = -1, 0, 1, 2, \cdots$, such that φ_{-1} has support in $[0, 1[$, φ_j has support in $\left]2^{j-1}, 2^{j+1}\right[$, $j \geq 0$, and that

$$\begin{aligned} \varphi_j(t) &= 1 si \left|t - 2^j\right| \leq \frac{1}{4} 2^j; \qquad j \geq 0, \\ \varphi_j^{(k)}(t) &\leq C_k 2^{jk}, \qquad j \geq 0. \end{aligned}$$

Such partition exists. Set

$$a_j(x, \xi) = \varphi_j \left(C \left(1 + |\xi|^2\right)^{\frac{\delta}{2}} \right) a(x, \xi),$$

for some constant $C > 0$.

Next, we aim to apply lemma (3.3).

Firstly, we estimate the norm of the operator $a_j(x, D)$. On the support of $a_j(x, \xi)$, we have

$$2^{j-1} \leq C \left(1 + |\xi|^2\right)^{\frac{\delta}{2}} \leq 2^j.$$

As a consequence, (3.1) yields

$$\text{(3.3)} \qquad \underset{x \in K}{Sup} \left| D_\xi^\alpha D_x^\beta a_j(x, \xi) \right| \leq C_0 \acute{C}_1^{|\alpha+\beta|} (|\alpha|!)^s (|\beta|!)^s \left(1 + |\xi|^2\right)^{j(|\beta| - |\alpha|)},$$

where $\acute{C}$ is a positive constant depending on j. Now, consider the unitary opertaor U_j on $L^2(\mathbb{R}^n)$, defined by

$$U_j \psi(x) = 2^{\frac{nj}{2}} \psi(2^j x)$$

It follows that $B_j = U_j^* A_j U_j$ is a pseudo-differential operator of Gevrey symbol type $b_j(x, \xi) = a_j(2^{-j} x, 2^j \xi)$, of class s, and (3.3) implies

$$\text{(3.4)} \qquad \underset{x \in K}{Sup} \left| D_\xi^\alpha D_x^\beta b_j(x, \xi) \right| \leq C_0 \acute{C}_1^{|\alpha+\beta|} (|\alpha|!)^s (|\beta|!)^s$$

Theorem (2.1) yields

$$\|A_j\| \leq CH,$$

where

$$H = \underset{|\alpha|, |\beta| \leq N}{Sup} C_0 \acute{C}_1^{|\alpha+\beta|} (|\alpha|!)^s (|\beta|!)^s.$$

Now, we give estimates of the norms of the operators $A_k^* A_j$ and $A_j A_k^*$, with $|k-j| \geq 4$. In each case, the symbols of A_j and A_k have disjoint supports, and $A_k^* A_j$ and $A_j A_k^*$ admit regular kernels. Hence, we may expect to obtain convenient bounds for their norms by elementary tools.

For $k-j \geq 4$, if $a_k(x,\eta)\ a_j(y,\xi) \neq 0$, then

$$\left(1+|\eta|^2\right)^{\frac{\delta}{2}} \sim 2^k \quad \text{and} \quad \left(1+|\xi|^2\right)^{\frac{\delta}{2}} \sim 2^j$$

and simultaneously, this implies

$$|\xi-\eta| \geq C(2^j+2^k)^{1+\gamma}\left(1+|\xi-\eta|^2\right)^{\frac{\gamma}{2}}, \quad \text{with} \quad \gamma = \frac{1-\delta}{\delta(1+\delta)} \tag{3.5}$$

Now,

$$A_k^* A_j u(x) = \int F(x,y)u(y)dy$$

where

$$F(x,y) = \int \overline{a_k(x,\xi)} a_j(z,\eta) e^{i(x\xi - z\xi + z\eta - y\eta)} dz d\xi d\eta$$

An integration by parts gives

$$F(x,y) = \int b_L(x,y,z,\xi,\eta) e^{i(x\xi - z\xi + z\eta - y\eta)} dz d\xi d\eta, \tag{3.6}$$

with

$$b_L(x,y,z,\xi,\eta) = \left(1+|x-z|^2\right)^{-L}\left(1+|z-y|^2\right)^{-L} \times$$
$$(1-\Delta_\xi)^N (1-\Delta_\eta)^L |\xi-\eta|^{2L} (-\Delta_z)^L \overline{a_k(z,\xi)} a_j(z,\eta).$$

Then

$$|b_L(x,y,z,\xi,\eta)| \leq C\left[\left(1+|x-z|^2\right)^{-\frac{L}{2}}\left(1+|z-y|^2\right)^{-\frac{L}{2}} \times |\xi-\eta|^{-1}(2^j+2^k)\right]^{2L}. \tag{3.7}$$

So, in Suppb_L, if $|k-j| \geq 4$, the relation (2.5) is plausible. Substituting into (2.7), yields

$$|b_L(x,y,z,\xi,\eta)| \leq C\left[\left(1+|x-z|^2\right)^{-\frac{L}{2}}\left(1+|z-y|^2\right)^{-\frac{L}{2}} \times \left(1+|\xi-\eta|^2\right)^{-\frac{\gamma}{2}}(2^j+2^k)^{-\gamma}\right]^{2L}. \tag{3.8}$$

If

$$L > Max\left(\frac{n}{2}, \frac{3}{2\gamma}, \frac{n}{2\gamma}\right),$$

we may make an integration in (3.8), and by (3.6), we obtain

$$|F(x,y)| \le C\left(1 + |x-y|^2\right)^{-L-\frac{\gamma}{2}} (2^j + 2^k)^{-\gamma},$$

which implies that

$$\|A_k^* A_j\| \le C(2^j + 2^k)^{-\gamma},$$

provided $|k-j| \ge 4$. Now, for $|k-j| \le 4$, we have

$$\|A_k^* A_j\| \le \|A_k^*\| \, \|A_j\| \le H^2.$$

Then, in all cases, we obtain

$$\|A_k^* A_j\| \le C 2^{-\gamma|j-k|}. \tag{3.9}$$

Estimating $A_j A_k^*$ is easier; indeed,

$$\widehat{A_j \; A_k^* u}(\xi) = \int \chi(\xi,\eta)\widehat{u}(\eta) d\eta,$$

where

$$\chi(x,\eta) = \int a_j(x,\zeta)\overline{a_k(y,\zeta)} e^{i(-x\xi + x\zeta - y\zeta + y\eta)} dx d\zeta dy.$$

Now, $|k-j| \ge 4$ implies $\chi(x,\eta) = 0$, then $A_j A_k^* = 0$ for $|k-j| \ge 4$. When $|k-j| \le 4$, we make use the inequality

$$\|A_k^* A_j\| \le \|A_k^*\| \, \|A_j\| \le H^2$$

to get

$$\|A_j A_k^*\| \le C 2^{-\gamma|j-k|}. \tag{3.10}$$

Combining (3.9) and (3.10) together with Cotlar-Knapp-Stein lemma, we deduce that the operator $A = a(x,D) = \sum A_j$ is bounded in $L^2(\mathbb{R}^n)$.

The second statement of the Theorem 3.1 is straightforward by estimating the constant C in (3.8), (3.9) and (3.10) by means of (3.1).

REFERENCES

1. R. Beals, *A general calculus of pseudo-differential operators*, Comm. Pure Appl. Math. **27** (1974), pp. 1-24.
2. A. Boulkhemair, Opérateurs pseudo-différentiels, paradifférentiels et intégraux de Fourier non réguliers: continuité et applications aux équations non linéaires, Thèse d'habilitation, Université de Nantes, 1998.
3. L. Boutet De Monvel and P. Krée, *Pseudo-differential operators and Gevrey classes*, Ann. Inst. Fourier, **17**, (1967), pp. 295 - 323.
4. A. Calderon et R. Vaillancourt, *A class of bounded pseudo-differential operators*, Proc. Nat. Acad. Sci. USA, **69** (1972), pp.1185-1187.
5. R. Coifman et Y. Meyer, *Au delà des opérateurs pseudo-différentiels*, Astérisque, **57** (1978).
6. G. De Donno and L. Rodino, *Gevrey hypoellipticity for partial differential equations with characteristics of higher multiplicity*, Rend. Scien. Mat. Univ. Pol. Torino, **58** (2000), pp. 435-448.
7. M. Hazi, *About the Gevrey regularity of the solutions of Euler's equation*, Arab Gulf Journal of scientific research, 16 (1998), pp. 611-620.
8. I.L. Hwang, *On the L^2-boundedness of pseudo-differential operators*, Trans. Amer. Math. Soc., **302** (1987), pp. 55-76.
9. P. Marcolongo, *On the solvability in Gevrey classes of linear operator in two variables*, Rend. Scien. Mat. Univ. Pol. Torino, **60** (2002), pp. 401-414.
10. T. Matsuzawa, *Journées d'équations aux dérivées partielles*, Saint Jean de Monts, 7 Juin 1982.
11. L. Rodino, *Linear partial differential operators in Gevrey classes*, World Scientific, 1993, Singapour.
12. M. E. Taylor, *Pseudo-differential operators*, Princeton University Press, Princeton, New Jersey, 1981.

Department of Mathematics,
Ecole Normale Supérieure
Kouba-Algiers, Algeria.
e-mail: hazi@ens-kouba.dz; hazi@hotmail.fr

Date received August 9, 2004