Arab J. Math. Sc.
Volume 6, Number 2, December 2000
pp. 17-26

SOME PROPERTIES OF α-IRRESOLUTE MULTIFUNCTIONS

VALERIU POPA AND TAKASHI NOIRI

ABSTRACT. Recently, some characterizations and several properties concerning upper (lower) α-irresolute multifunctions are introduced and studied in [9]. In this paper, further characterizations and properties of upper and lower α-irresolute multifunctions are obtained.

1. INTRODUCTION

In 1965, Njåstad [6] introduced a weak form of open sets called α-sets. In 1980, Maheshwari and Thakur [4] defined a function from a topological space into a topological space to be α-irresolute if the inverse image of each α-set is an α-set. Recently, Noiri and Nasef [9] have defined a multifunction $F : X \to Y$ to be upper (lower) α-irresolute if $F^+(V)$ (resp. $F^-(V)$) is an α-set for each α-set V of Y. Some characterizations and several properties concerning upper (lower) α-irresolute multifunctions are studied in [9]. The purpose of the present paper is to obtain further characterizations and properties of upper (lower) α-irresolute multifunctions.

2. PRELIMINARIES

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A is said to be *α-open* (or *α-set*) [6] (resp. *semi-open* [3], *preopen* [5]) if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(A))$). The family of all α-open sets (resp. semi-open, preopen) sets in (X, τ) is denoted by $\alpha(X)$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$). For these three families, it is shown in [7, Lemma 3.1] that

Mathematics Subject Classification. 54C60.

$\alpha(X) = \mathrm{SO}(X) \cap \mathrm{PO}(X)$. The family of all α-open sets of X containing a point $x \in X$ is denoted by $\alpha(X,x)$. Since $\alpha(X)$ is a topology for X [6, Proposition 2], by $\alpha\mathrm{Cl}(A)$ (resp. $\alpha\mathrm{Int}(A)$) we shall denote the closure (resp. interior) of A with respect to $\alpha(X)$. The complement of a semi-open set is said to be *semi-closed* [1]. The intersection of all semi-closed sets of X containing A is called the *semi-closure* of A [1] and is denoted by $\mathrm{sCl}(A)$.
Throughout this paper, spaces (X,τ) and (Y,σ) (or simply X and Y) always mean topological spaces and $F : X \to Y$ presents a multivalued function. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set G of Y by $F^+(G)$ and $F^-(G)$, respectively, that is

$$F^+(G) = \{x \in X : F(x) \subset G\} and F^-(G) = \{x \in X : F(x) \cap G \neq \emptyset\}.$$

Definition 1. A multifunction $F : X \to Y$ is said to be
(a) *upper α-irresolute* [9] at a point $x \in X$ if for each α-open set V containing $F(x)$, there exists $U \in \alpha(X,x)$ such that $F(U) \subset V$,
(b) *lower α-irresolute* [9] at a point $x \in X$ if for each α-open set V such that $F(x) \cap V \neq \emptyset$, there exists $U \in \alpha(X,x)$ such that $F(u) \cap V \neq \emptyset$ for every $u \in U$,
(c) *upper (lower) α-irresolute* if F has this property at every point of X.

3. CHARACTERIZATIONS

In this section we obtain further characterizations of upper and lower α-irresolute multifunctions.

Definition 2. A subset A of a topological space X is said to be *α-regular* [2] (resp. *α^*-regular*) if for any point $a \in A$ and any open (resp. α-open) set U containing a, there exists an open set G of X such that $a \in G \subset \mathrm{Cl}(G) \subset U$, or equivalently if for each closed (resp. α-closed) set W and each point $a \in A$ such that $a \notin W$, there exists two disjoint open sets U and V such that $a \in U$ and $W \subset V$.

Remark. Every α^*-regular set is α-regular.

Definition 3. A subset A of a topological space X is said to be *α-paracompact* [2] if every cover of A by open sets of X has an X-open X-locally finite refinement which covers A.

Lemma 1. *If A is an α^*-regular α-paracompact subset of a topological space X and $A \subset U \in \alpha(X)$, then there exists an open set G of X such that $A \subset G \subset \mathrm{Cl}(G) \subset U$.*

Proof. The proof is similar to one of [2, Theorem 2.5] and is thus omitted.

Definition 4. A multifunction $F : X \to Y$ is said to be *punctually α-paracompact* (resp. *punctually α^*-regular*) if for each point $x \in X$, $F(x)$ is α-paracompact (resp. α^*-regular).

Lemma 2. *If A is an α^*-regular set of a topological space X, then for every α-open set D which intersects A, there exists and open set D_A such that $A \cap D_A \neq \emptyset$ and $Cl(D_A) \subset D$.*

Proof. Let A be an α^*-regular set of X and D an α-open set such that $A \cap D \neq \emptyset$. Let $x \in A \cap D$. Then $x \in A$ and $x \in D$. Since $D \in \alpha(X)$, there exists an open set D_A such that $x \in D_A \subset \mathrm{Cl}(D_A) \subset D$. Therefore, we obtain $x \in A \cap D_A$ and $\mathrm{Cl}(D_A) \subset D$.

By $\alpha\mathrm{Cl}(F) : X \to Y$ [13, p.483], we shall denote a multifunction defined as follows: $[\alpha\mathrm{Cl}(F)](x) = \alpha\mathrm{Cl}(F(x))$ for each $x \in X$.

Lemma 3. *If $F : X \to Y$ is a punctually α^*-regular and punctually α-paracompact, then $[\alpha\mathrm{Cl}(F)]^+(V) = F^+(V)$ for each $V \in \alpha(Y)$.*

Proof. Let V be any α-open set of Y and $x \in [\alpha\mathrm{Cl}(F)]^+(V)$. Then $\alpha\mathrm{Cl}(F(x)) \subset V$ and $F(x) \subset V$. Therefore, $x \in F^+(V)$ and $[\alpha\mathrm{Cl}(F)]^+(V) \subset F^+(V)$. Conversely, let V be any α-open set of Y and $x \in F^+(V)$. Then $F(x) \subset V$. Since $F(x)$ is α^*-regular and α-paracompact, by Lemma 1 there exists an open set G such that $F(x) \subset G \subset \mathrm{Cl}(G) \subset V$; hence $\alpha\mathrm{Cl}(F(x)) \subset \mathrm{Cl}(G) \subset V$. This shows that $x \in [\alpha\mathrm{Cl}(F)]^+(V)$ and hence $F^+(V) \subset [\alpha\ \mathrm{Cl}(F)]^+(V)$.

Theorem 4. *Let $F : X \to Y$ be punctually α^*-regular and punctually α-paracompact. Then F is upper α-irresolute if and only if $\alpha\mathrm{Cl}(F) : X \to Y$ is upper α-irresolute.*

Proof. *Necessity.* Suppose that F is upper α-irresolute. Let $x \in X$ and V be any α-open set of Y such that $\alpha\mathrm{Cl}(F)(x) \subset V$. By Lemma 3, we have $x \in [\alpha\mathrm{Cl}(F)]^+(V) = F^+(V)$. Since F is upper α-irresolute, there exists $U \in \alpha(X, x)$ such that $F(U) \subset V$. Since $F(u)$ is α^*-regular and α-paracompact for each $u \in U$, by Lemma 1 there exists an open set H such that $F(u) \subset H \subset \mathrm{Cl}(H) \subset V$. Therefore, we have $\alpha\mathrm{Cl}(F(u)) \subset \mathrm{Cl}(H) \subset V$ for each $u \in U$ and hence $\alpha\mathrm{Cl}(F)(U) \subset V$. This shows that $\alpha\mathrm{Cl}(F)$ is upper α-irresolute.

Sufficiency. Suppose that $\alpha\mathrm{Cl}(F) : X \to Y$ is upper α-irresolute. Let $x \in X$ and V be any α-open set of Y such that $F(x) \subset V$. By Lemma 3, we have $x \in F^+(V) = [\alpha\mathrm{Cl}(F)]^+(V)$ and hence $[\alpha\mathrm{Cl}(F)](x) \subset V$. Since $\alpha\mathrm{Cl}(F)$ is upper α-irresolute, there exists $U \in \alpha(X, x)$ such that $[\alpha\mathrm{Cl}(F)](U) \subset V$; hence $F(U) \subset V$. This shows that F is upper α-irresolute.

Lemma 5. (Popa and Noiri [13]). *If $F : X \to Y$ is a multifunction, then $[\alpha\mathrm{Cl}(F)]^-(V) = F^-(V)$ for every $V \in \alpha(Y)$.*

Theorem 6. *A multifunction $F : X \to Y$ is lower α-irresolute if and only if $\alpha\mathrm{Cl}(F) : X \to Y$ is lower α-irresolute.*

Proof. Necessity. Suppose that F is lower α-irresolute. Let $x \in X$ and V be any α-open set of Y such that $[\alpha\mathrm{Cl}(F)](x) \cap V \neq \emptyset$. By Lemma 4, we have $x \in [\alpha\mathrm{Cl}(F)]^-(V) = F^-(V)$ and $F(x) \cap V \neq \emptyset$. Since F is lower α-irresolute, there exists $U \in \alpha(X, x)$ such that $F(u) \cap V \neq \emptyset$ for every $u \in U$. Therefore, we have $[\alpha\mathrm{Cl}(F)](u) \cap V \neq \emptyset$ for every $u \in U$. It follows that $\alpha\mathrm{Cl}(F)$ is lower α-irresolute.

Sufficiency. Suppose that $\alpha\mathrm{Cl}(F)$ is lower α-irresolute. Let $x \in X$ and V be any α-open set of Y such that $F(x) \cap V \neq \emptyset$. By Lemma 4, we have $x \in F^-(V) = [\alpha\mathrm{Cl}(F)]^-(V)$ and hence $[\alpha\mathrm{Cl}(F)](x) \cap V \neq \emptyset$. Since $\alpha\mathrm{Cl}(F)$ is lower α-irresolute at x, there exists $U \in \alpha(X, x)$ such that $[\alpha\mathrm{Cl}(F)](u) \cap V \neq \emptyset$ for every $u \in U$. Since V is α-open in Y, we obtain $F(u) \cap V \neq \emptyset$ for every $u \in U$. It follows that F is lower α-irresolute.

Lemma 7 (Popa and Noiri [13]). *Let A and B be subsets of a topological space X.*

(a) If $A \in \mathrm{SO}(X) \cup \mathrm{PO}(X)$ and $B \in \alpha(X)$, then $A \cap B \in \alpha(A)$.

(b) If $A \subset B \subset X, A \in \alpha(B)$ and $B \in \alpha(X)$, then $A \in \alpha(X)$.

Theorem 8. *If a multifunction $F : X \to Y$ is upper α-irresolute (resp. lower α-irresolute) and $X_0 \in \mathrm{PO}(X) \cup \mathrm{SO}(X)$, then the restriction $F|_{X_0} : X_0 \to Y$ is upper α-irresolute (resp. lower α-irresolute).*

Proof. We prove only the assertion for F is upper α-irresolute, the proof for F lower α-irresolute being analogous. Let $x \in X_0$ and $V \in \alpha(Y)$ such that $(F|_{X_0})(x) \subset V$. Since F is upper α-irresolute and $(F|_{X_0})(x) = F(x)$, there exists $U \in \alpha(X, x)$ such that $F(U) \subset V$. Set $U_0 = U \cap X_0$, then by Lemma 5 we have $x \in U_0 \in \alpha(X, x)$ and $(F|_{X_0})(U_0) \subset V$. This shows that $F|_{X_0}$ is upper α-irresolute.

Theorem 9. *A multifunction $F : X \to Y$ is upper α-irresolute (resp. lower α-irresolute) if for each $x \in X$ there exists $X_0 \in \alpha(X, x)$ such that the restriction $F|_{X_0} : X_0 \to Y$ is upper α-irresolute (resp. lower α-irresolute).*

Proof. We prove only the assertion for F is upper α-irresolute, the proof for F lower α-irresolute being analogous. Let $x \in X$ and $V \in \alpha(Y)$ such that $F(x) \subset V$. There exists $X_0 \in \alpha(X, x)$ such that $F|_{X_0} : X_0 \to Y$ is upper α-irresolute. Since $(F|_{X_0})(x) = F(x) \subset V$, there exists $U_0 \in \alpha(X_0, x)$ such that $(F|X_0)(U_0) \subset V$. By Lemma 5, $U_0 \in \alpha(X)$ and $F(u) = (F|_{X_0})(u)$ for every $u \in U_0$. This shows that F is upper α-irresolute.

Corollary 10. *Let $\{ U_\gamma : \gamma \in \Gamma \}$ be an α-open cover of X. A multifunction $F : X \to Y$ is upper α-irresolute (resp. lower α-irresolute) if and only if the restriction $F|_{U_\gamma} : U_\gamma \to Y$ is upper α-irresolute (resp. lower α-irresolute) for each $\gamma \in \Gamma$.*

Proof. This is an immediate consequence of Theorems 3 and 4.

4. SOME PROPERTIES

Definition 5. A function $F : X \to Y$ is said to be

(a) *upper almost weakly continuous* (*u.a.w.c.*) [8] if for each $x \in X$ and each open set V containing $F(x)$, $x \in Int(Cl(F^+(Cl(V))))$,

(b) *lower almost weakly continuous* (*l.a.w.c.*) [8] if for each $x \in X$ and each open set V such that $F(x) \cap V \neq \emptyset$, $x \in Int(Cl(F^-(Cl(V))))$.

Definition 6. For a multifunction $F : X \to Y$, we put

$$A^+(F) = \{x \in X : F \text{ is u.a.w.c. at } x\},$$

$$A^-(F) = \{x \in X : F \text{ is l.a.w.c. at } x\},$$

$$\alpha^+(F) = \{x \in X : F \text{ is upper } \alpha\text{-irresolute at } x\},$$

$$\alpha^-(F) = \{x \in X : F \text{ is lower } \alpha\text{-irresolute at } x\}, \text{ and}$$

$$\alpha(F) = \alpha^+(F) \cap \alpha^-(F).$$

Lemma 11 (Noiri and Nasef [9]). *A multifunction $F : X \to Y$ is upper α-irresolute (resp. lower α-irresolute) at $x \in X$ if and only if for any $U \in \mathrm{SO}(X, x)$ and any $V \in \alpha(Y)$ such that $F(x) \subset V$ (resp. $F(x) \cap V \neq \emptyset$), there exists a nonempty open set U_V of X such that $U_V \subset U$ and $F(U_V) \subset V$ (resp. $F(u) \cap V \neq \emptyset$ for every $u \in U_V$).*

Theorem 12. *If a multifunction $F : X \to Y$ is punctually α^*-regular and punctually α-paracompact, then $A^+(F) \cap \mathrm{sCl}(\alpha(F)) \subset \alpha^+(F)$.*

Proof. Suppose that $x \in A^+(F) \cap \mathrm{sCl}(\alpha(F))$. Let U be a semi-open set of X containing x and $V \in \alpha(Y)$ such that $F(x) \subset V$. Since $F(x)$ is α^*-regular and α-paracompact, by Lemma 1 there exists an open set W such that $F(x) \subset W \subset \mathrm{Cl}(W) \subset V$. Since F is u.a.w.c. at x and $F(x) \subset W$, there exists an open set U_1 such that $x \in U_1 \subset \mathrm{Cl}(F^+(\mathrm{Cl}(W)))$. Put $G = U \cap U_1$, then G is a semi-open set, $x \in G \subset \mathrm{Cl}(F^+(\mathrm{Cl}(W)))$ and $G \cap \alpha(F) \neq \emptyset$. Therefore, we obtain the following property:

(1) If $z \in G \cap \alpha(F)$, then $z \in F^+(\mathrm{Cl}(W))$.

Suppose that (1) does not hold. Then there exists $z \in G \cap \alpha(F)$ such that $z \in F^-(Y - \mathrm{Cl}(W))$. Since $z \in \alpha(F)$, then F is lower α-irresolute at z and by Lemma 6 there exists a nonempty open set $G_1 \subset G$ such that $G_1 \subset F^-(Y - \mathrm{Cl}(W)) = X - F^+(\mathrm{Cl}(W))$. Then, we have $G_1 \cap F^+(\mathrm{Cl}(W)) = \emptyset$ and hence $G_1 \cap \mathrm{Cl}(F^+(\mathrm{Cl}(W))) = \emptyset$ since G_1 is open. This contradicts that $G_1 \subset G \subset \mathrm{Cl}(F^+(\mathrm{Cl}(W)))$. Therefore, (1) holds. By (1) it follows that

$F(z) \subset \mathrm{Cl}(W) \subset V$. Since $z \in \alpha(F)$, F is upper α-irresolute at z and by Lemma 6 there exists a nonempty open set $H \subset U$ such that $F(H) \subset V$. Thus, it follows from Lemma 6 that F is upper α-irresolute at x.

Theorem 13. *If a multifunction $F : X \to Y$ is punctually α^*-regular, then $A^-(F) \cap \mathrm{sCl}(\alpha(F)) \subset \alpha^-(F)$.*

Proof. Suppose that $x \in A^-(F) \cap \mathrm{sCl}(\alpha(F))$. Let U be a semi-open set of X containing x and $V \in \alpha(Y)$ such that $F(x) \cap V \neq \emptyset$. Since $F(x)$ is α^*-regular, by Lemma 2 there exists an open set W such that $F(x) \cap W \neq \emptyset$ and $\mathrm{Cl}(W) \subset V$. Since F is l.a.w.c. at x and $F(x) \cap W \neq \emptyset$, there exists an open set U_1 such that $x \in U_1 \subset \mathrm{Cl}(F^-(\mathrm{Cl}(W)))$. Put $G = U \cap U_1$, then G is semi-open in X, $x \in G \subset \mathrm{Cl}(F^-(\mathrm{Cl}(W)))$ and $G \cap \alpha(F) \neq \emptyset$. Therefore, we obtain the following property:

(2) If $z \in G \cap \alpha(F)$, then $z \in F^-(\mathrm{Cl}(W))$.

Suppose that (2) does not hold. Then, there exists $z \in G \cap \alpha(F)$ such that $z \in F^+(Y - \mathrm{Cl}(W))$. Since $z \in \alpha(F)$, then F is upper α-irresolute at z and it follows from Lemma 6 that there exists a nonempty open set $G_1 \subset G$ such that $G_1 \subset F^+(Y - \mathrm{Cl}(W)) = X - F^-(\mathrm{Cl}(W))$. Then, we have $G_1 \cap F^-(\mathrm{Cl}(W)) = \emptyset$ and hence $G_1 \cap \mathrm{Cl}(F^-(\mathrm{Cl}(W))) = \emptyset$ since G_1 is open. This contradicts that $G_1 \subset G \subset \mathrm{Cl}(F^-(\mathrm{Cl}(W)))$. Therefore, (2) holds. By (2), it follows that $F(z) \cap \mathrm{Cl}(W)) \neq \emptyset$ and hence $F(z) \cap V \neq \emptyset$. Since $z \in \alpha(F)$, F is lower α-irresolute at z and by Lemma 6 there exists an nonempty open set $H \subset U$ such that $H \subset F^-(V)$. Thus, it follows from Lemma 6 that F is lower α-irresolute at x.

Corollary 14. *Let $F : X \to Y$ be an u.a.w.c. and l.a.w.c. multifunction. If F is punctually α^*-regular and punctually α-paracompact, then $\alpha(F)$ is semi-closed in X.*

Proof. This is an immediate consequence of Theorems 5 and 6.

Definition 7. A mutifunction $F : X \to Y$ is said to be

(a) *upper weakly continuous* [10] if for each $x \in X$ and each open set V containing $F(x)$, there exists an open neighbourhood U of x such that

$F(U) \subset Cl(V)$,

(b) *lower weakly continuous* [10] if for each $x \in X$ and each open set V such that $F(x) \cap V \neq \emptyset$, there exists an open neighbourhood U of x such that $F(u) \cap Cl(V) \neq \emptyset$ for every $u \in U$.

Definition 8. A mutifunction $F : X \to Y$ is said to be

(a) *upper H-almost continuous* [11] if for each $x \in X$ and each open set V containing $F(x)$, $x \in Int(Cl(F^+(V)))$,

(b) *lower H-almost continuous* [11] if for each $x \in X$ and each open set V such that $F(x) \cap V \neq \emptyset$, $x \in Int(Cl(F^-(V)))$.

Theorem 15. *Let $F : X \to Y$ be an upper and lower H-almost continuous multifunction. If F is punctually α^*-regular and punctually α-paracompact and $sCl(\alpha(F)) = X$, then F is continuous.*

Proof. Since every upper (resp. lower) H-almost continuous multifunction is u.a.w.c. (resp. l.a.w.c.), by Corollary 2 we have $\alpha(F) = \mathrm{sCl}(\alpha(F)) = X$. It follows from [13, Theorems 4.3 and 4.4] that F is upper and lower weakly continuous. Moreover, since every α^*-regular set is α-regular, it follows from [12, Theorems 1 and 2] that F is upper semi-continuous and lower semi-continuous. Thus F is continuous.

Definition 9. The α-frontier of a subset A of a space X, denoted $\alpha\mathrm{Fr}(A)$, is defined by $\alpha\mathrm{Fr}(A)=\alpha\mathrm{Cl}(A)\cap\alpha\mathrm{Cl}(X-A)=\alpha\mathrm{Cl}(A)-\alpha\mathrm{Int}(A)$.

Theorem 16. *The set of all points x of X at which a multifunction $F : X \to Y$ is not upper α-irresolute (resp. lower α-irresolute) is identical with the union of the α-frontier of the upper (resp. lower) inverse image of α-open sets containing (resp. meeting) $F(x)$.*

Proof. Let x be a point of X at which $F(x)$ is not upper α-irresolute. Then, there exists $V \in \alpha(Y)$ containing $F(x)$ such that $U \cap (X - F^+(V)) \neq \emptyset$ for every $U \in \alpha(X, x)$. By [4, Lemma 3], $x \in \alpha\mathrm{Cl}(X - F^+(V))$. Since $x \in F^+(V)$, we have $x \in \alpha\mathrm{Cl}(F^+(V))$ and hence $x \in \alpha\mathrm{Fr}(F^+(V))$. Conversely, suppose that $V \in \alpha(Y)$ containing $F(x)$ and let $x \in \alpha\mathrm{Fr}(F^+(V))$. If F is upper α-

irresolute at x, then there exists $U \in \alpha(X, x)$ such that $F(U) \subset V$; hence $U \subset F^+(V)$. Therefore, we obtain $x \in U \subset \alpha\text{Int}(F^+(V))$. This contradicts that $x \in \alpha\text{Fr}(F^+(V))$. Thus F is not upper α-irresolute at x. The case for lower α-irresolute is similarly shown.

REFERENCES

1. S. G. Crossley and S. K. Hildebrand, *Semi-closure*, Texas J. Sci. 22 (1971), 99–112.
2. I. Kovačević, *Subsets and paracompactness*, Univ. u Novom Sadu Zb. Rad. Prirod.-Mat. Fak. Ser. Mat. 14 (1984), 79–87.
3. N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly 70 (1963), 36–41.
4. S. N. Maheshwari and S. S. Thakur, *On α-irresolute mappings*, Tamkang J. Math. 11 (1980), 209–214.
5. A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt 53 (1982), 47–53.
6. O. Njåstad, *On some classes of nearly open sets*, Pacific J. Math. 15 (1965), 961-970.
7. T. Noiri, *On α-continuous functions*, Časopis Pěst. Mat. 109 (1984), 118–126.
8. T. Noiri and V. Popa, *Almost weakly continuous multifunctions* , Demonstratio Math. 26 (1993), 363–380.
9. T. Noiri and A. Nasef, *On upper and lower $\alpha\alpha$-irresolute multifunctions*, Res. Rep. Yatsushiro Nat. Coll. Tech. 20 (1998), 105–110.
10. V. Popa, *Weakly continuous multifunctions*, Boll. Un. Mat. Ital. (5) 15-A (1978), 379–388.
11. V. Popa, *On certain properties of quasicontinuous and almost continuous multifunctions* (Romanian), Stud. Cerc. Mat. 30 (1978), 441–446.

12. V. Popa, *A note on weakly and almost continuous multifunctions* , Univ. u Novom Sadu Zb. Rad. Prirod.-Mat. Fak. Ser. Mat. 21 (1991), 31–38.

13. V. Popa and T. Noiri, *On upper and lower α -continuous multifunctions*, Math. Slovaca 43 (1993), 477–491.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF BACAU, 5500 BACAU, ROMANIA.
E-MAIL: VPOPA@UB.RO

DEPARTMENT OF MATHEMATICS, YATSUSHIRO COLLEGE OF TECHNOLOGY, YATSUSHIRO, KUMAMOTO, 866-8501, JAPAN.
E-MAIL: NOIRI@AS.YATSUSHIRO-NCT.AC.JP

Date received March 21, 1999.