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ODD NONLINEAR BINOMIAL STATES: STATISTICAL PROPERTIES

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Abstract. In this paper an odd nonlinear binomial state is introduced. It interpolates between the odd number state and the odd nonlinear coherent state. Some statistical properties, such as the Glauber second-order correlation function and squeezing phenomenon (amplitude-squared squeezing) for this state are discussed. The quasiprobability distribution functions (Husimi Qfunction and Wigner W-function) are also examined. The phase probability distribution function and the quadrature distribution are discussed.

Keywords. Nonlinear states, Nonclassical effects, Quasidistribution functions.

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1. Introduction

The most familiar state for the electromagnetic field is the Fock number state $|n\rangle$, which is an eigenstate of the photon number operator $\widehat{n} = \widehat{a}^{\dagger}\widehat{a}$, i.e ($\widehat{a}^{\dagger}\widehat{a}|n\rangle = n|n\rangle$ where $\widehat{a}$ and $\widehat{a}^{\dagger}$ are the annihilation and creation operators of the electromagnetic field, respectively), which satisfy $[\widehat{a}, \widehat{a}^{\dagger}] = 1$. Fock states can be produced by applying the operator $\widehat{a}^{\dagger n}$ on the vacuum state $|0\rangle$ such that $|n\rangle = \frac{\widehat{a}^{\dagger}}{\sqrt{n!}}|0\rangle$. On the other hand,the coherent state —$|n\rangle$, can be obtained by applying the displacement operator $\widehat{D}(\alpha) = \exp(\widehat{a}^{*}\alpha - \alpha^{*}\widehat{a})$ on the vacuum

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state where ξ is a complex amplitude [1, 2]. Furthermore, the construction of intermediate states such as the binomial states (the states which interpolate between coherent states and number states) is also a linear combination from the number states with coefficients chosen such that the photon counting probability distribution is a binomial distribution. The binomial state (BS) $|\xi, M\rangle$ is a finite linear combination of number states defined as [3, 4]:

$$|M,\xi\rangle = \sum_{n=0}^{M} \sqrt{\binom{M}{n}} \xi^n (1-|\xi|^2)^{\frac{M-n}{2}} |n\rangle, \tag{1.1}$$

where M is a non-negative integer, ξ is a complex number $(0 < |\xi| < 1)$ and $|n\rangle$ is a number state of the radiation field. The photon number distribution is clearly a binomial distribution, whence the name binomial state. The notion of BS was also generalized to the intermediate number-squeezed states [5] and the numberphase states [6], as well as their q-deformation [7]. The nonlinear binomial state, which interpolates between the nonlinear coherent and number states was introduced with a generation scheme for the state, which is defined by [8]

$$|M,\xi\rangle_f = \lambda_f \sum_{n=0}^{M} \sqrt{\binom{M}{n}} \xi^n (1-|\xi|^2)^{\frac{M-n}{2}} (f(n))!|n\rangle, \tag{1.2}$$

where λ_f is the normalization constant given by

$$|\lambda_f|^{-2} = \sum_{n=0}^{M} \binom{M}{n} \xi^{2n} (1-|\xi|^2)^{M-n} [(f(n))!]^2, \tag{1.3}$$

with ξ a complex number and $f(n)$ a well behaved function of the integer n, and $f(n)! = f(n)f(n-1)......f(2)f(1)f(0), f(0) \neq 0$. In addition, the nonlinear negative binomial state was introduced and some of its properties have been discussed [9]. The paper is organized as follows. In Section 2 the odd nonlinear binomial states of the radiation field as superposition of a pair of nonlinear BS is introduced. These states can give as limiting cases the odd nonlinear coherent states. In Section 3 we concentrate on the nonclassical effects more precisely on squeezing phenomenon as well as on super-Poisonian and sub-Poissoinan behavior by using the Glauber second-order correlation function. Furthermore, we discuss the quasi-probability distribution functions (Q-function and Wigner function), the phase probability distribution function and the position distribution function. Finally, we close the paper with our conclusion in Section 4.

2. Odd nonlinear binomial states (ONLBS).

One can introduce the ONLBS as a superposition of nonlinear binomial states in the form

$$|M,\xi\rangle_0 = \sum_{n=0}^{[\frac{M-1}{2}]} C(2n+1)|2n+1\rangle, \tag{2.1}$$

with

$$C(2n+1) = \lambda_0 \binom{M}{2n+1}^{\frac{1}{2}} \xi^{(2n+1)} (1-|\xi|^2)^{\frac{M-2n-1}{2}} (f(2n+1))! \tag{2.2}$$

and λ_0 is the normalization constant given by

$$|\lambda_0|^{-2} = \sum_{n=0}^{[\frac{M-1}{2}]} \binom{M}{2n+1} |\xi|^{2(2n+1)} (1-|\xi|^2)^{\frac{M-2n-1}{2}} [(f(2n+1))!]^2 \tag{2.3}$$

These states are superposition of the two nonlinear binomial states $|M,\xi\rangle$ and $|M,-\xi\rangle$. As a matter of fact $|M,-\xi\rangle_0 \propto (|M,\xi\rangle - |M,-\xi\rangle)$. The ONLBS are states composed of odd Fock state only. When the $(+)$ sign is taken instead of the $(-)$ sign, another state results. It is compressed of even Fock state only, and hence called even nonlinear binomial state ENLBS. Such superposition can have nonclassical properties. These states (ONLBS and ENLBS) reduce to the odd binomial states and even binomial states when the nonlinearity function $f(n)$ is taken to be unity [10]. Furthermore in the limit $M \longrightarrow \infty, \xi \longrightarrow 0$ and $M|\xi|^2 \longrightarrow$ finite number, these states reduce to odd (even) coherent states with their nonclassical properties even when nonlinearities are excluded [11]. The nonlinear binomial states can be used to produce the odd (even) nonlinear state by the method introduced by Yurke Stoler to produce superposition of coherent states [12]. For this purpose, we have to allow the nonlinear binomial states to propagate through an amplitude dispersive medium. Under suitable conditions we can obtain a quantum superposition of two NLBS. Another method is to allow the NLBS to interact with a Kere-like medium in one arm of a Mech-Zehdner interferometer and a resonant two-level atom crossing one of the outports of the same interferometer. This methodology was reported for the possibility of generaliting even and odd coherent states [13]. Since the BS represents a coherent state as a limiting condition of its parameters, hence for certain parameters one can adopt this method for generating the proposed

states.
In what follows we consider some of the nonclassical properties of these states.

3. Non-classical properties.

In this section we shall study the non-classical properties of the ONLBS. In the present work the nonlinearity function will be taken as two forms, the first form is $f(n) = \frac{1}{\sqrt{n}}$ and $f(0) = 1$ [8, 9, 14]. A choice of this type appears in a natural way in Hamiltonians describing interaction with intensity-dependent coupling between a two level atom and the electromagnetic field [15, 21]. While the second form is [22, 25]:

$$f(n) = L_n^1(\eta^2)[(n+1)L_n^0(\eta^2)]^{-1} \tag{3.1}$$

with

$$L_n^k(x) = \sum_{m=0}^{n}(-1)^m \frac{(n+k)!x^m}{(n-m)!(k+m)!m!} \tag{3.2}$$

where $L_n^k(x)$ are the associated Laguerre polynomials and is known as the Lamb-Dicke parameter. This function (3.1) appears in the study of the quantized motion of the center of mass of trapped ions [26].

3.1. **Second-order correlation function.** We shall employ the Glauber second-order correlation function to discuss some statistical properties such as sub-Poissonian distribution [27, 28] which is characteristic of nonclassical states. The condition for sub-Poissonian statistics, for any state, is that the variance $\langle(\Delta\widehat{n})^2\rangle = \langle\widehat{n}^2\rangle - \langle\widehat{n}\rangle^2$ must be less than the mean photon number $\langle\widehat{n}\rangle$. This can be presented by the Glauber (zero time) second-order correlation function of the form

$$g^2(0) = \frac{\langle a^{\dagger 2}a^2\rangle}{\langle a^\dagger a\rangle^2} = 1 + \frac{\langle(\Delta\widehat{n})^2\rangle - \langle\widehat{n}\rangle}{\langle\widehat{n}\rangle^2} \tag{3.3}$$

A light field has a sub-Poissonian distribution (a nonclassical effect) if $g^{(2)}(0) < 1$, super-Poissonian distribution if $g^{(2)}(0) > 1$, and Poissonian distribution if $g^{(2)}(0) = 1$. Examples for sub-Poissonian, super-Poissonian and Poissonian light statistics are Fock, chaotic and coherent states, respectively. Moreover, the generation of sub-Poissonian light has been established in semiconductor laser [29] and in the microwave region using maser operating in the microscopic regime [30]. To discuss$g^{(2)}(0)$ of the present states, we should calculate the expectation values of $\langle\widehat{n}^r\rangle = \langle(\widehat{a}^\dagger\widehat{a})^r\rangle$ with respect to these states, which read as follows

$$\langle\widehat{n}^r\rangle = \sum_{n=0}^{[\frac{M-1}{2}]} [C(2n+1)]^2(2n+1)^r \tag{3.4}$$

we plot $g^{(2)}(0)$ as a function of ξ for different values of M and η in Figs.(1). In (Fig. 1a), the state with form 1 starts sub-Poissonian and pursues this behavior for the all values of M considered and the whole range of ξ. The dependence on the parameter η (form 2) of the Laguerre polynomials shows at increasing η with $M = 10$ shifts of the sub-Poissonian a little higher for the state as shown in Fig. 1b.

3.2. **Amplitude squared squeezing (ASS).** It is well known that squeezing phenomenon represents one of the interesting phenomena in the field of quantum optics. The phenomenon is a direct quantum effect of the Heisenberg uncertainty principle which reflects the reduced quantum fluctuations in one of the field quadratures at the expense of the other corresponding stretched quadrature. Squeezed light is related to optical waveguide tap [31, 32], also to interferometric techniques [33], and to several applications in optical communication networks [34, 35]. Here we concentrate on amplitude squared squeezing [36, 37]. This kind of squeezing arises in a natural way in the second harmonic generation, and its quadratures component are defined as

$$\widehat{d_1} = \frac{1}{2}(\widehat{a}^2 + \widehat{a}^{\dagger 2}), \tag{3.5}$$

$$\widehat{d_2} = \frac{i}{2}(\widehat{a}^2 - \widehat{a}^{\dagger 2}). \tag{3.6}$$

Then $\widehat{d_1}$ and $\widehat{d_2}$ obey the commutation relation

$$[\widehat{d_1}, \widehat{d_2}] = \frac{1}{2}i[\widehat{a}^2, \widehat{a}^{\dagger 2}] = i(2\widehat{n}+1), \tag{3.7}$$

while their quadrature variances satisfy the uncertainty relation

$$\langle(\Delta\widehat{d_1})^2\rangle\langle(\Delta\widehat{d_2})^2\rangle \geq |\langle\widehat{n}+\frac{1}{2}\rangle|^2. \tag{3.8}$$

The ONLBS is said to be ASS if one of the quadratures $\Delta\widehat{d_1^2}$ or $\langle(\Delta\widehat{d_1})^2\rangle \leq |\langle\widehat{n}+\frac{1}{2}\rangle|$. This means that

$$\Delta\widehat{d_1^2} = Re\langle\widehat{a}^4\rangle + \langle\widehat{n}^2\rangle - \langle\widehat{n}\rangle - 2Re(\langle\widehat{a}^2\rangle)^2 < 0 \tag{3.9}$$

$$\Delta\widehat{d_2^2} = \langle\widehat{n}^2\rangle - \langle\widehat{n}\rangle - Re\langle\widehat{a}^4\rangle + 2Im(\langle\widehat{a}\rangle)^2 < 0 \tag{3.10}$$

For the present states, we have obtained the following expectation values of the operators $\widehat{a}^s$:

$$\langle\widehat{a}^s\rangle = \sum_{j=0}^{[\frac{M-s-1}{2}]} [C(2j+1)][C(2j+s+1)]\sqrt{\frac{(2j+s+1)!}{(2j+1)!}} \tag{3.11}$$

From the above equations, we have investigated the ASS for the ONLBS. Now $\Delta\widehat{d_1^2}$ and $\Delta\widehat{d_2^2}$ are computed and the results are presented in Figs.(2). For the case of form 1, we see that ASS is shown for almost the whole range of ξ for the state. By increasing M stronger squeezing occurs as can be seen in fig. (2a). While, when we consider the second form of the nonlinearity we see that ASS occurs for an interval of the range of ξ and then it is revoked for higher values of ξ. The range of squeezing depends on M and η. For the same M increasing b shortens the range of squeezing (see Fig. 2b). Similarly for the same η increasing M may shorten the range of squeezing (see Fig. 2c). For the quadrature squeezing, we may note that the expectation values for any odd power of $\widehat{a}$ or $\widehat{a}^\dagger$ will vanish. Therefore when we write $x = \frac{1}{2}(\widehat{a}+\widehat{a}^{\dagger 2})$ and $y = \frac{1}{2i}(\widehat{a}-\widehat{a}^{\dagger 2})$ we find that $\Delta^2 x = \frac{1}{4}(\langle\widehat{a}^2\rangle + \langle\widehat{a}\dagger 2\rangle + 2\langle\widehat{n}\rangle + 1)$ which will not show any squeezing in this quadrature but $\Delta^2 y = \frac{1}{4}(2\langle\widehat{n}\rangle + 1 - \langle\widehat{a}^2\rangle - \langle\widehat{a}^{\dagger 2}\rangle)$ may show squeezing.

3.3. **Quasi-probability distribution functions.** Now let us continue our progress and devote the present section to consider and discuss the quasi-probability distribution functions. It is well known that there are three types of these functions: Glauber-Sudarshan P, Wigner-Moyal W, and Husimi-Kano Qfunctions corresponding to normally ordered, symmetrically, and anti-normally ordered, respectively [38, 41]. These functions are important tools to give insight on the statistical description of a quantum mechanical system. For experimental point of view these functions can be measured via homodyne tomography [42]. In order to evaluate these functions we need to calculate the characteristic function for the desired states. The s-parameterized quasi-probability function for the state $|\Psi\rangle$ can be cast bin a series representation in the form [43]

$$F(\alpha,s)=\frac{2}{\pi(1-s)}\sum_{n=0}^{\infty}(\frac{s+1}{s-1})^{n}|\langle\Psi|\widehat{D}(\alpha)|n\rangle|^{2} \tag{3.12}$$

where $\widehat{D}(\alpha)$ is the Glauber displacement operator $\widehat{D}(\alpha)=\exp(\widehat{a}^{*}\alpha-\alpha^{*}\widehat{a})$. For the state (2.1), we find that

$$F(\alpha,s)=\frac{2\exp(\frac{-2|\alpha|^{2}}{1-s})}{\pi(1-s)}\sum_{m,n=0}^{\frac{M-1}{2}}C^{*}(2m+1)C(2n+1)$$

$$(\frac{n!}{m!})^{1/2}(\frac{2}{1-s})^{m-n}(\frac{s+1}{s-1})^{n}L_{n}^{m-n}(\frac{4|\alpha|^{2}}{1-s^{2}}) \tag{3.13}$$

The s-parameterized function $F(\alpha,s)$ gives the W-Wigner function when s is taken to be zero, the Q-function when s tends to one. The P-function is highly singular for the Fock states therefore we shall not discuss it here. For other representations such as the non-diagonal P-representation and positive P-representation we may refer to the ref. [44]. We shall concentrate on the Q-function and W-function only. Q-function is calculated for the present states in the following

$$Q(\alpha)=\pi^{-1}\exp(-|\alpha|^{2})|G|^{2} \tag{3.14}$$

where

$$G=\sum_{n=0}^{[\frac{M-1}{2}]}C(2n+1)\frac{(\alpha^{*})^{2n+1}}{\sqrt{(2n+1)!}} \tag{3.15}$$

with $\alpha=x+iy$.

Curves depicting the Q-function for the states are found in Figs.(3). The ONLBS shows splitting even for small values of ξ as shown in fig. 3a. The effect of the Laguerre polynomials function shows its effects on the Q-function even for small values of η (when $\eta=0.2$). While when $M=10$ the change over the form 1 is not clear. The state shows the dependence on η much clearer and more apparent. The dependence on η exhibits more distinct changes in the state as can be seen from Figs.(3c, d). The double peak structure is a consequence of the superposition. This structure and interference pattern is characteristic for such states constructed from combination of mesoscopic states. The case of the nonlinear binomial state as shown in [8] shows only a single peak structure. However, W-function can take negative values for some states and this is regarded as reflection of nonclassical effects. The W-function ($W(\alpha)$) with the off-diagonal terms taken into account in the case of the present states is defined as follows

$$\begin{aligned} W(\alpha) &= \frac{2}{\pi}\exp(-2|\alpha|^2)\Big[\sum_{n=0}^{[\frac{M-1}{2}]}(-1)^{(2n+1)}|C(2n+1)|^2 L_{2n}^{0}(4|\alpha|^2) \\ &+ \sum_{r=1}^{[\frac{M-1}{2}]}\sum_{n=0}^{r-1}(-1)^{(2n+1)}\overline{C}(2r+1)C(2n+1)\sqrt{\frac{(2n+1)!}{(2r+1)!}} \\ &\times L_{2n+1}^{2(r-n)}(4|\alpha|^2)[(2\alpha^*)^{2(r+n+1)}+(2\alpha)^{2(r-n)}]\Big] \end{aligned} \tag{3.16}$$

In Figs.(4), we have plotted the W-function for different values of M, ξ and η Non-classical effects can be studied by measuring the negative values of the W-function. These non-classical effects are exhibited in these Figs. of the W-function for the present states. The state has a negative spike at the origin due to the inclusion of the odd states only. In Fig. 4a, the effect of the 1st exited state is dominant for small value of $\xi = 0.2$ and moderate $M = 10$. By increasing the value of M or the value of ξ the effects of higher excitations are included as wobbles appear surrounding the central spike (Fig. 4b). Noting that non-classicality is apparent due to the negative values that the function attains. The effect of the second form due to the dependence on the associated Laguerre polynomials is exhibited clearly in the Figs. of the Wigner functions (Figs. 4c). The oscillations of these polynomials increase due to the effect of the higher excitations (Fig. 4d). As it is observed more wobbles are shown surrounding the central spikes. Increasing the value of the parameter $M = 15$ leads to a decrease in the amplitude values of these wobbles for the state Fig.(4e). Due to the existence of the odd Fock states alone in the ONLBS, these states have negative values at the origan and consequently a very pronounced signature of non-classicality. Therefore these states are more convenient to measure nonclassical effects. Furthermore the interference patterns are more pronounced than their counterparts in the case of the NLBS [8].

3.4. **Phase distribution.** The notion of the phase in quantum optics has found renewed interest because of the existence of phase-dependent quantum noise. In this section, the phase properties using the Pegg-Barnett method [45, 46] are studied. This method is based on the phase states $|\Theta_m\rangle$, which are defined as

$$|\Theta_m\rangle = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(in\theta_m)|n\rangle, \tag{3.17}$$

where

$$\theta_m = \theta_0 + \frac{2\pi m}{s+1}; m = 0, 1, ..., s. \tag{3.18}$$

The value of θ_0 is arbitrary. The set $\{|\Theta_m\rangle\}$ indicates a specific bases set of $(s+1)$ mutually orthogonal phase states. In fact the phase states $|\Theta_m\rangle$ are eigenstates of the Hermitian phase operator $\widehat{\Phi}_\theta$ given by

$$\widehat{\Phi}_\theta = \sum_{m=0}^{s} \theta_m |\Theta_m\rangle\langle\Theta_m| \tag{3.19}$$

The expectation values are calculated in the finite dimensional space and after that the limit $s \longrightarrow \infty$ is taken. The state of the form

$$|b\rangle = \sum_{m=0}^{s} b_n e^{in\Psi}|n\rangle \tag{3.20}$$

is called a partial phase state [47], where b_n are real and positive and Ψ is a phase. From equations (3.17) and (3.20), one can calculate the expectation values of the phase operator and its moments. However, we shall concentrate on the phase probability distribution. This distribution for the partial phase state of equation (3.20) is given by

$$Ph(\theta) = |\langle\Theta_m|b\rangle|^2 \tag{3.21}$$

Since the density of phase states is $\frac{s+1}{2\pi}$, thus in the continuum limit $s \longrightarrow \infty$ equation (3.21) reduces to

$$Ph(\xi, \theta) = \frac{1}{2\pi}\Big[1 + 2\sum_{n>m}\sum_{m} b_m b_n \cos[(n-m)\theta]\Big] \tag{3.22}$$

In what follows one calculates this function for the present states. It is found to be

$$Ph(\xi, \theta) = \frac{1}{2\pi}\Big[1 + 2\sum_{r=1}^{[\frac{M-1}{2}]}\sum_{s=0}^{r-1} \overline{C}(2s+1)C(2r+1)\cos[2(s-r)\theta]\Big] \tag{3.23}$$

In figures (5), the Pegg-Barnett phase distribution $Ph(\xi, \theta)$ given by equation (3.23) is plotted against the parameters ξ and θ, for different values of M and

η. It is to be noted that as i increases the state moves from the Fock state $|1\rangle$, Fig.(5a) at $\xi = 0$ to the full state for larger ξ. Thus for $\xi = 0$ the phase information is lost and as ξ increases the phase is built up to end up as a single peak at $theta = 0$ and two wings at $\pm\pi$. These wings do not appear in the case of the non linear binomial states, only the peak at $\theta = 0$ appears there [8]. The nonlinearity functional dependence shows its effects as in the oscillations of the peaks and their heights for different values of b as shown in Figs.(5b).

3.5. **The quadrature distribution.** To investigate the quadrature (or position) distribution $P(x,\xi)$, which can be measured in the homodyne detector [42]. The quadrature distribution can be evaluated via the W-function through the relation

$$P(x,\xi) = \int_{-\infty}^{+\infty} W(\alpha,\xi)dy, \tag{3.24}$$

in this case if one inserts equation (3.16) into the above equation (3.24) and evaluates the integral, then we have

$$\begin{aligned} P(x,\xi) &= \sqrt{\frac{2}{\pi}} e^{-2x^2} \sum_{n=0}^{[\frac{M}{2}]} \frac{(-1)^{(2n)}}{2^n n!} \\ &\times |C(2r)|^2 |H_{2n}(\sqrt{2}x)|^2 \end{aligned} \tag{3.25}$$

where $H_k(z)$ is the Hermite polynomial of degree k defined by

$$H_k(z) = \sum_{n=0}^{[\frac{k}{2}]} \frac{(-1)^s k!(2z)^{(k-2s)}}{s!(k-2s)!}. \tag{3.26}$$

To demonstrate the behavior of the distribution function $P(x,\xi)$ we have plotted in Figs.(6) the function against the variable x. Because of the form of the function $P(x,\xi)$ we see that it takes negative values for the present states (Fig. 6a). On the other hand as ξ or M increases other oscillations around the negative central peak appears. These negative side peaks gain height by increasing ξ or M. For $P(x,\xi), x = 0$ and as $|x|$ increase it moves to minima and then increases until it reaches the value zero again. Increasing M lowers the minimum value while increasing ξ results in appearance of other oscillations. The effect of η lowers the minima of the peaks for the present states.

Conclusion. Odd nonlinear binomial state as a superposition of nonlinear binomial state has been presented. These states have been constructed and two forms have been chosen as the nonlinearity function. The effect of the nonlinearity function on the different characteristics of the states has been discussed. Non-classical properties such that, the second order correlation function and the amplitude squared squeezing have been discussed. The results show that ASS exist for these states. The quasi-probability distribution and the phase distribution functions for the two forms of the nonlinearity functions have been computed. Also the position distribution has been calculated. The forms of the nonlinearity functions affect the properties greatly. It has been shown that when we take the nonlinearity form depends on Laguerre polynomials, then the states exhibit different properties. Finally, these states may find applications in related fields of quantum optics and quantum information, due to their nonclassical properties.

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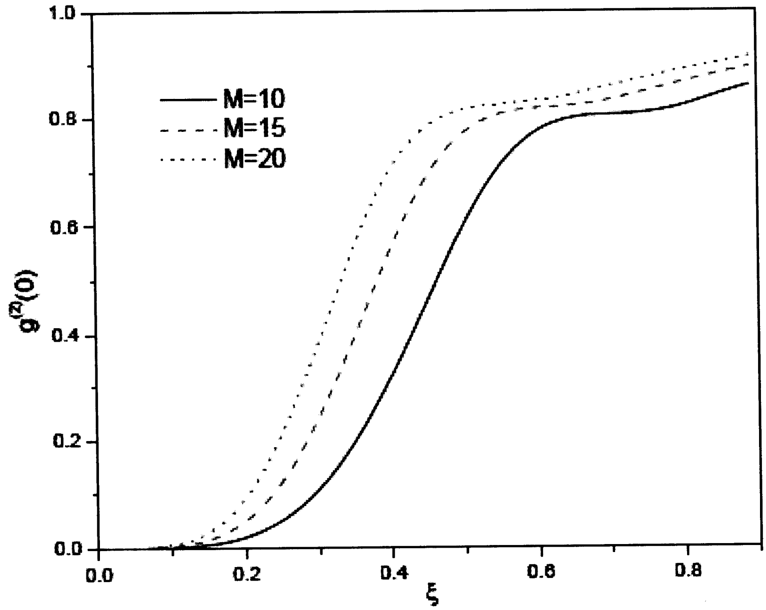

Fig 1a: $g^{(2)}(0)$ for form 1.

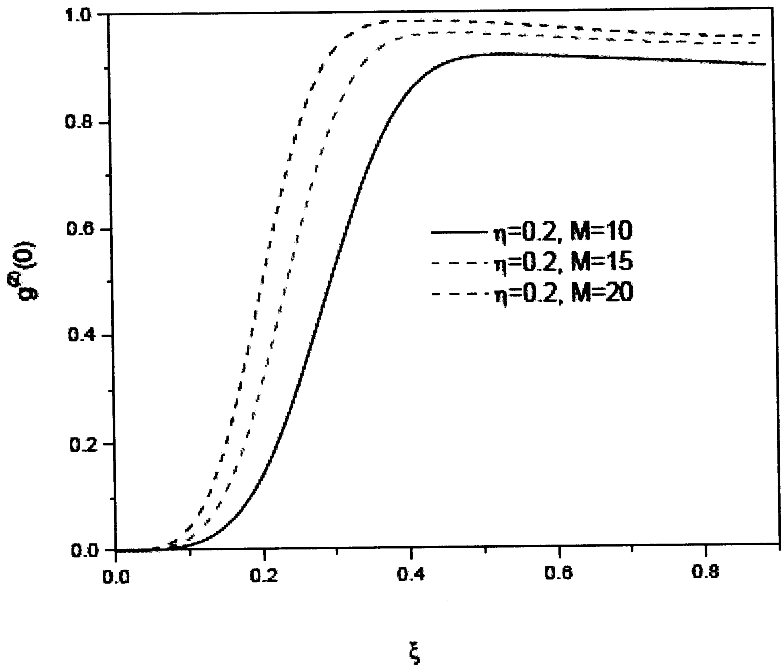

fig 1b: $g^{(2)}(0)$ for form 2.

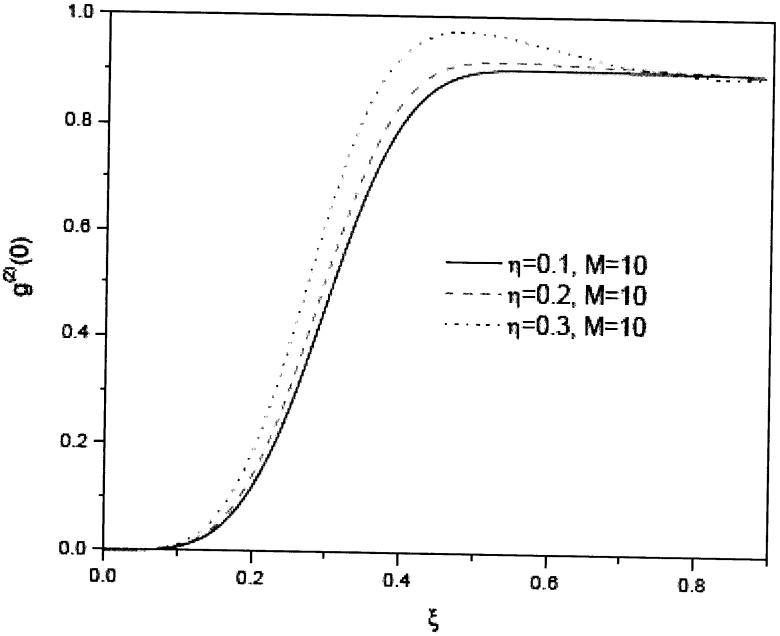

fig 1c: $g^{(2)}(0)$ for form 2.

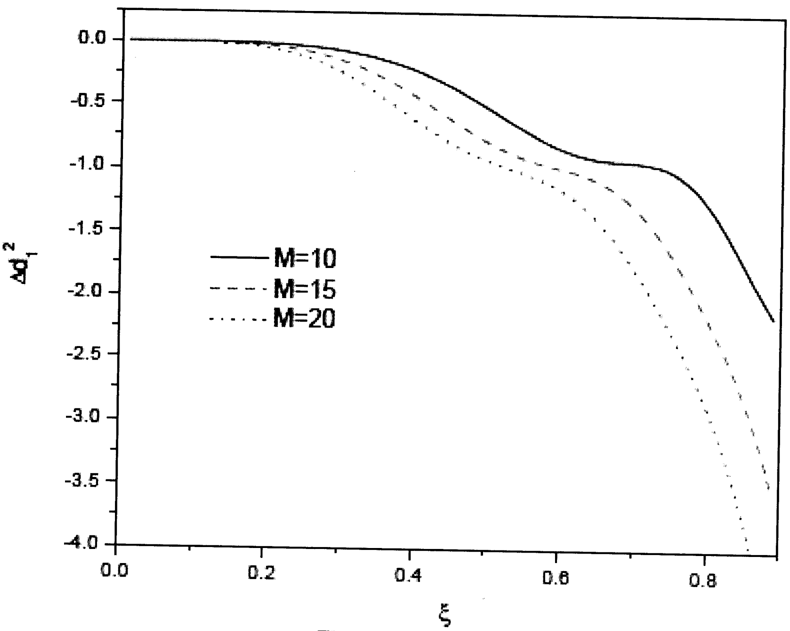

Fig 2a: ASS for form 1.

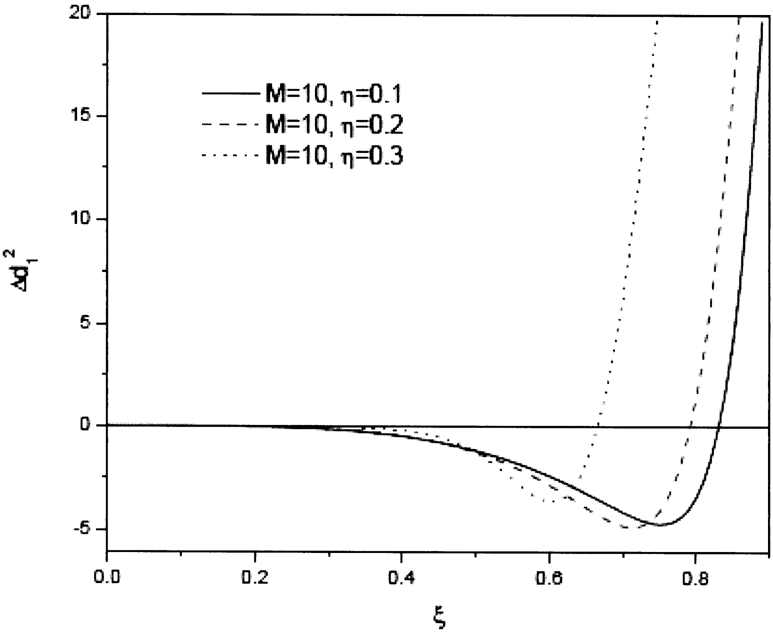

Fig 2b: ASS for form 2.

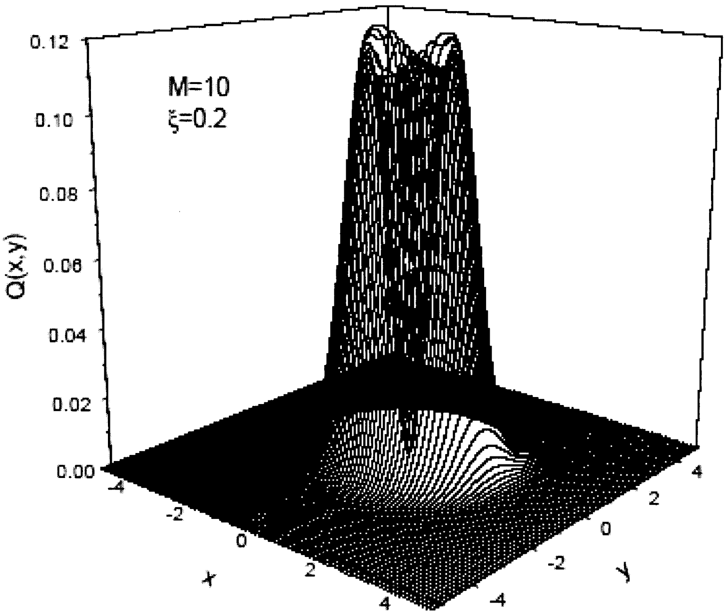

Fig 3a: Q- function for form 1.

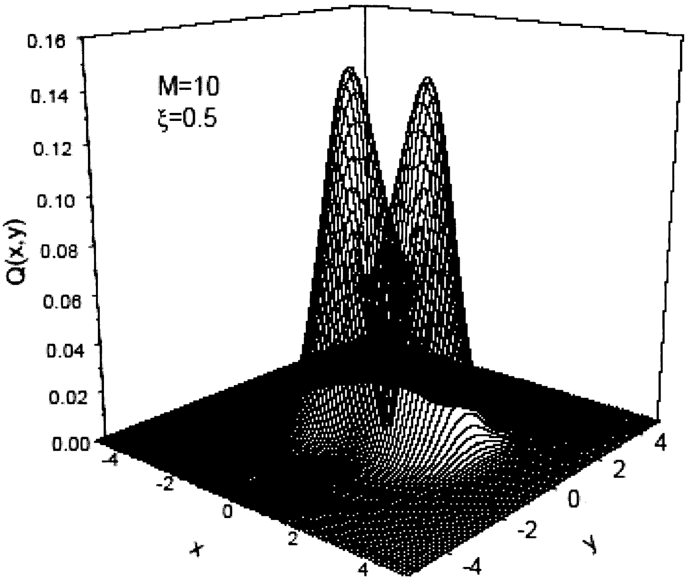

Fig 3b: Q- function for form 1.

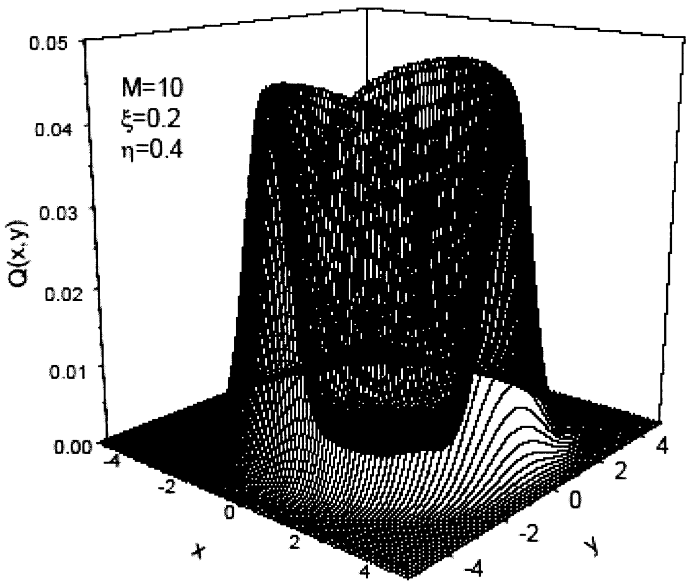

Fig 3c: Q-function for form 2.

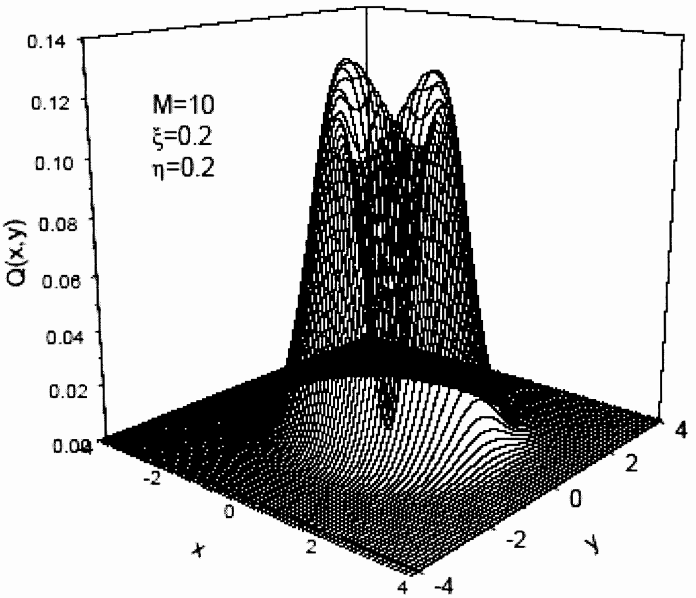

Fig 3d: Q-function for form 2.

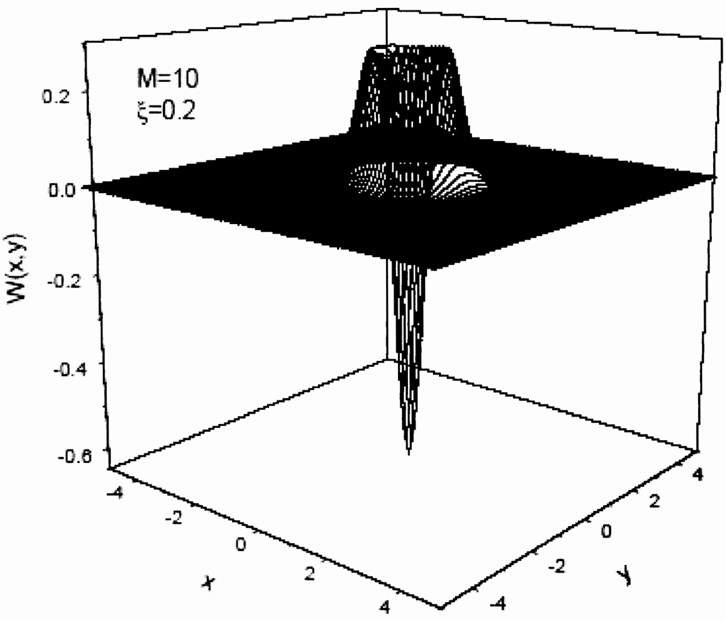

fig. 4a: Wigner function for form 1.

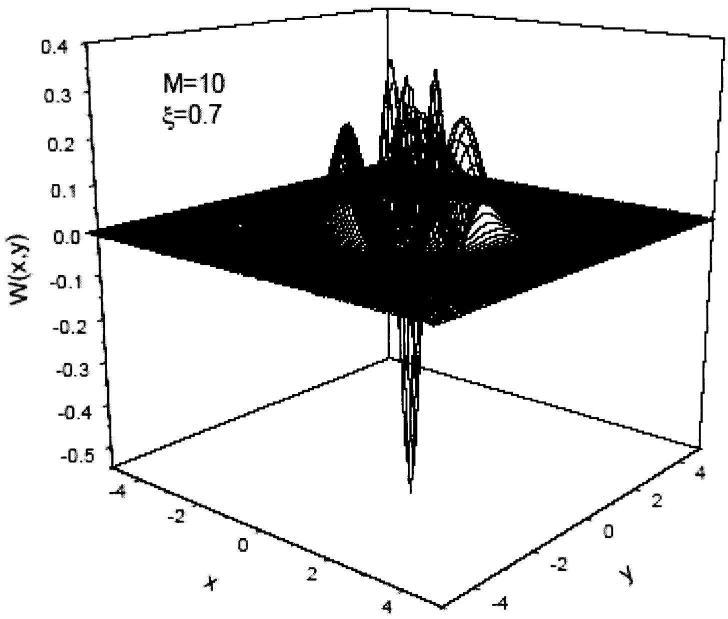

fig. 4b: Wigner function for form 1.

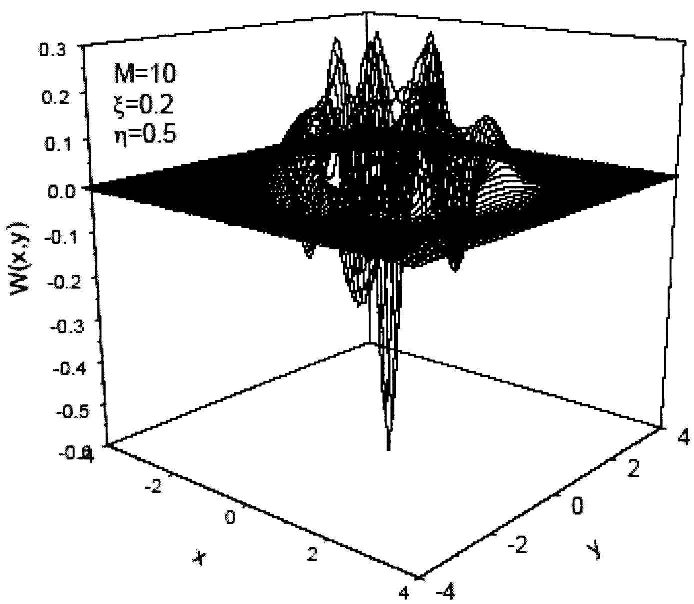

Fig. 4c: Wigner Function for form 2.

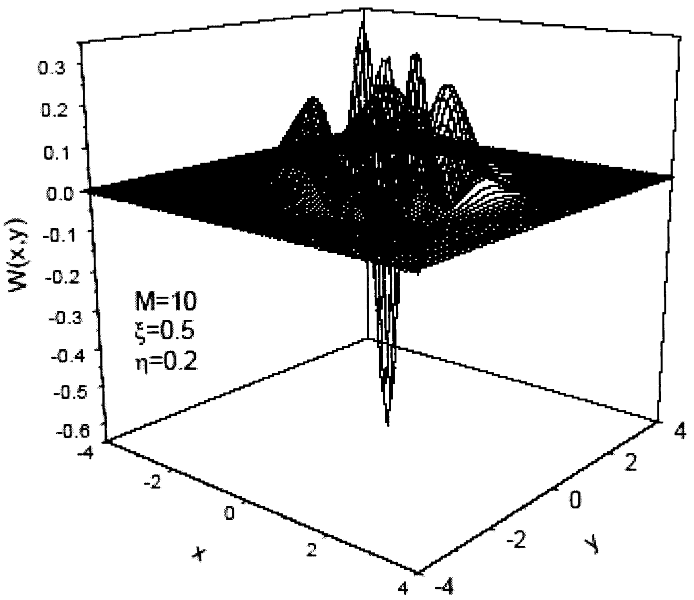

Fig. 4d: Wigner Function for form 2.

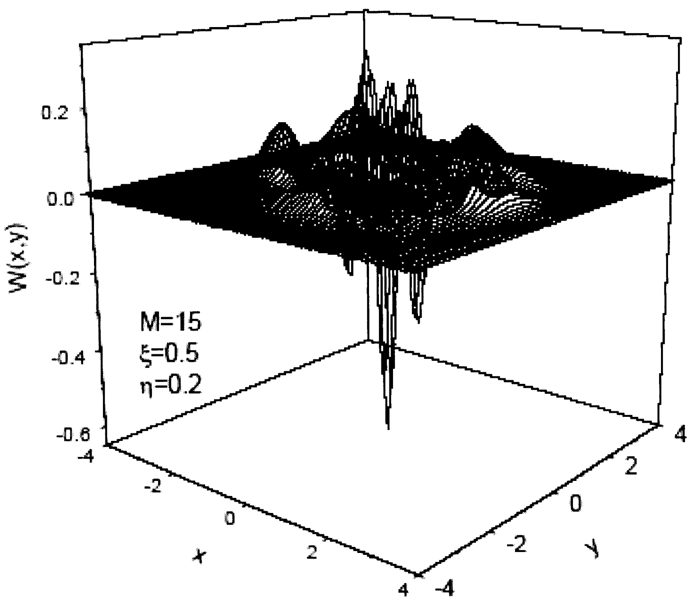

Fig. 4e: Wigner Function for form 2.

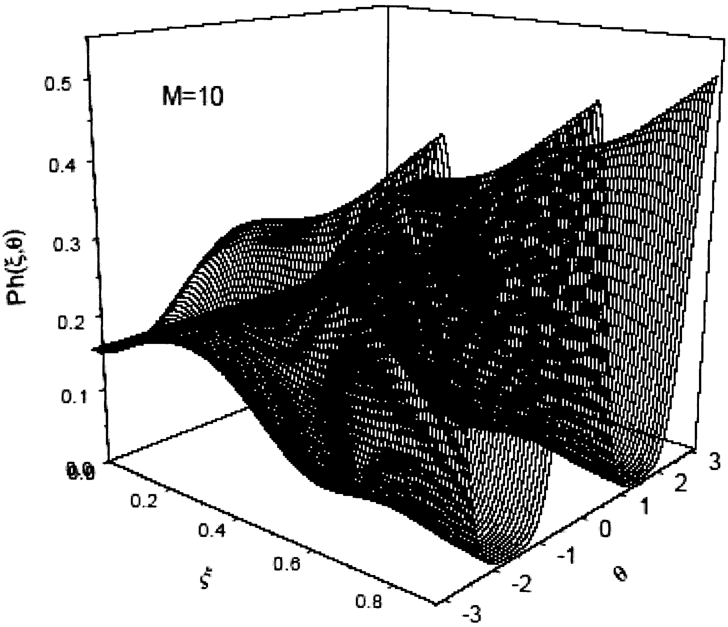

fig. 5a: phase distribution for form 1.

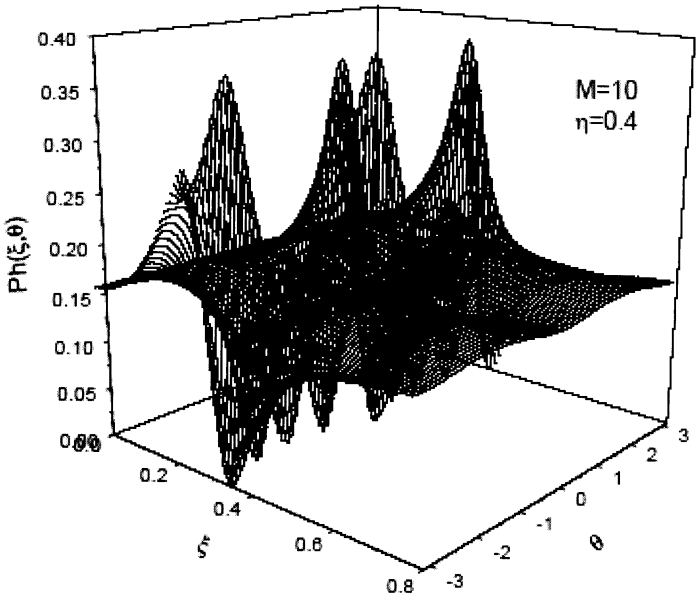

fig. 5b: phase distribution for form 2.

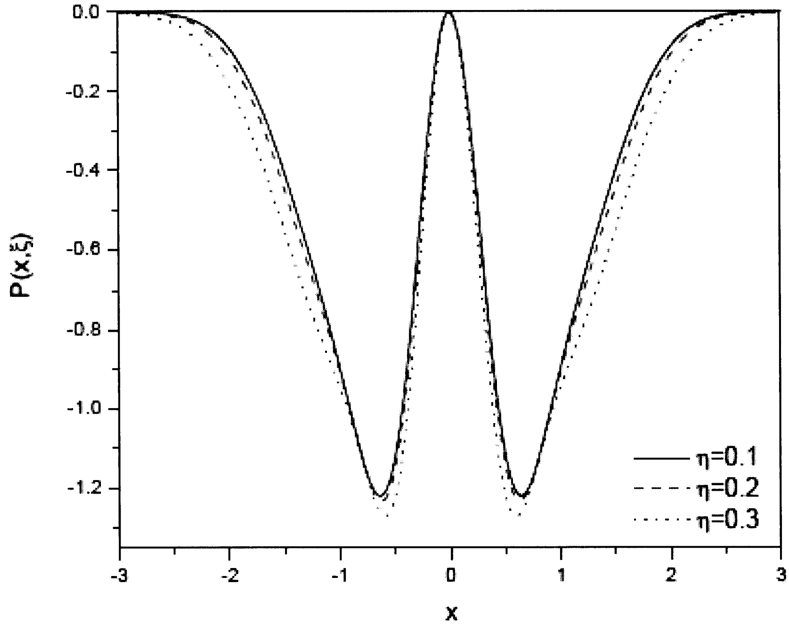

Fig. 6j: Quadrature distribution at M=10 and ξ=0.2 (form 2).

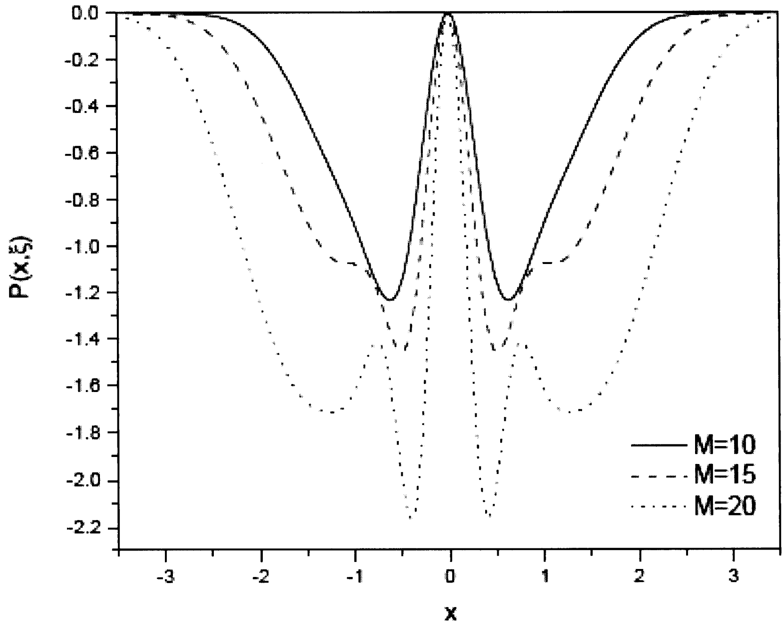

Fig. 6h: Quadrature distribution at ξ=η=0.2 (form 2).

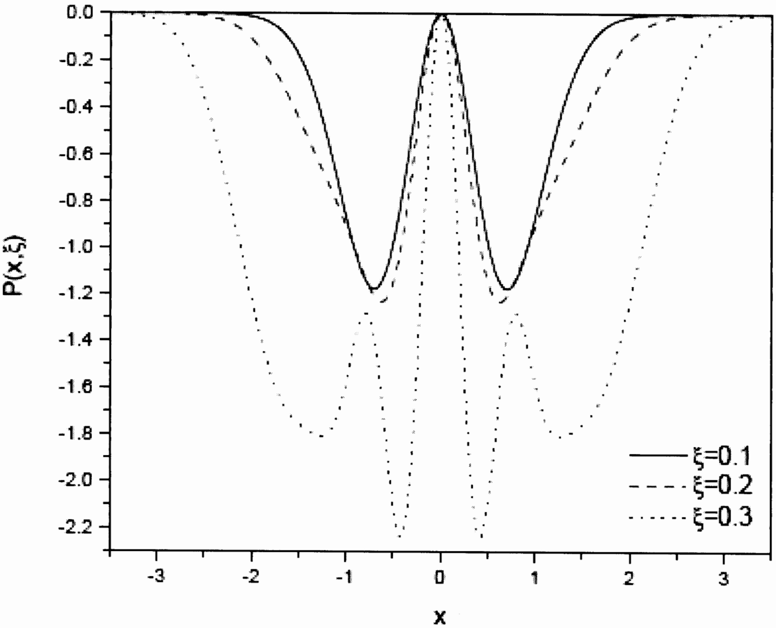

Fig. 6i: Quadrature distribution at M=10, h=0.2 (form 2).