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A THEOREM ON COMPLEX SUBMANIFOLDS OF A COMPLEX PROJECTIVE SPACE

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ABSTRACT. In this paper we study compact Kaehler submanifolds of a Complex projective space of constant holomorphic sectional curvature and obtain conditions under which these submanifolds are totally geodesic.

0. INTRODUCTION

The Theory of Kaehler submanifolds in a complex manifold is widely studied in differential geometry. In [2], The results on complex submanifolds were arranged by K. Ogiue. In [3], A. Ros proved that if every holomorphic sectional curvature of a Kaehlerian submanifold M is greater than $\frac{1}{2}$ then M is totally geodesic in CP^m. This result was conjectured by K. Ogiue in [2]. One of the interesting problems in the geometry of complex submanifolds is to find conditions on complex submanifolds to be totally geodesic. These conditions generally involve the pinching of the sectional curvatures or the scalar curvature.

Let M be a Riemannian manifold and $\overline{M}$ be an almost Hermitian manifold with almost complex structure J. An isometric immersion $f : M \to \overline{M}$ of M in $\overline{M}$ is called a holomorphic immersion if at each point $p \in M$ we have $J(T_p(M)) = T_pM$, where T_pM denotes the tangent space of M at p. Such a submanifold M of a Kaehlerian manifold $\overline{M}$ is called a Kaehlerian submanifold. It is well known that Kaehlerian submanifolds are minimal in a complex space form(or, a Kaehlerian manifold).

Recently S. Deshmukh [1] has obtained a new integral formula and he studied rigidity of minimal submanifolds of a unit sphere S^{n+p}.

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In this paper, we will study compact complex submanifolds of a complex projective space with scalar curvature satisfying the pinching condition $4r \geq c(4n^2+n+1)$, n being the complex dimension of the submanifold . We show that these submanifolds are totally geodesic under an additional condition. Finally, we note that we use the same approach as in [1].

1. Preliminaries

Let M be a Kaehler submanifold of complex dimension n, of a complex space form $\overline{M}^{n+m}(c)$.We denote by $\Gamma(TM)$ and $\Gamma\left(TM^{\perp}\right)$ the modules of differentiable sections of the tangent bundle TM and normal bundle $TM^{\perp}$. The Riemannian connection $\overline{\nabla}$ induces Riemannian connections ∇ and $\nabla^{\perp}$ on M and the normal bundle $TM^{\perp}$, respectively, satisfying

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{1.1}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp}_X Y \tag{1.2}$$

for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma\left(TM^{\perp}\right)$,where h and A_N are the second fundamental form and the Weingarten map respectively. The second fundamental form h and the shape operator A are related by

$$g(A_N X, Y) = g(h(X,Y), N). \tag{1.3}$$

Moreover, the second fundamental form h and the induced connection ∇ satisfy

$$\nabla_X JY = J\nabla_X Y, h(X, JY) = Jh(X,Y) = h(JX,Y) \tag{1.4}$$

or, equivalently,

$$JA_N X = -A_N JX = A_{JN} X \tag{1.5}$$

for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma\left(TM^{\perp}\right)$,(cf. [4]).

Let R and $R^{\perp}$ be the curvature tensors corresponding to the connections ∇ and $\nabla^{\perp}$ respectively. Then the equations of Gauss, Codazzi and Ricci are

$$\begin{aligned} R(X,Y)Z &= \frac{c}{4}\{g(Y,Z)X - g(X,Z)Y + g(JY,Z)JX \\ &\quad -g(JX,Z)JY + 2g(X,JY)JZ\} \\ &\quad +A_{h(Y,Z)}X - A_{h(X,Z)}Y \end{aligned} \tag{1.6}$$

$$(1.7) \qquad (\nabla_X h)(Y,Z) = (\nabla_Y h)(X,Z)$$

$$(1.8) \quad g(R^{\perp}(X,Y)N_1,N_2) = -g([A_{N_2},A_{N_1}]X,Y) + \frac{c}{2}g(X,JY)g(JN_1,N_2)$$

for X, Y, Z tangent to M and $N_1, N_2 \in \Gamma(TM^{\perp})$.

The covariant derivative $(\nabla_X h)(Y,Z)$ is given by

$$(\nabla_X h)(Y,Z) = (\nabla h)(X,Y,Z) = \nabla_X^{\perp} h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z).$$

From (1.6) the Ricci tensor Ric and scalar curvature r are (cf. [4]), respectively,

$$(1.9) \qquad Ric(X,Y) = \frac{1}{2}(n+1)cg(X,Y) - \sum_k g(h(X,e_k),h(Y,e_k)$$

$$(1.10) \qquad r = n(n+1)c - \sum_{ij} g(h(e_i,e_j),h(e_i,e_j))$$

where $\{e_1, ..., e_n, e_1^* = Je_1, ..., e_n^* = Je_n\}$ is a local orthonormal frame on M.

The Ricci operator Q is defined by

$$Ric(X,Y) = g(QX,Y).$$

Thus, from (1.9), we have

$$(1.11) \qquad QX = \frac{1}{2}(n+1)cX - \sum_i A_{h(e_i,X)}e_i.$$

We define the second derivative of h by

$$\begin{aligned}(\nabla^2 h)(X,Y,Z,W) &= \nabla_X^{\perp}(\nabla h)(Y,Z,W) - (\nabla h)(\nabla_X Y, Z, W) \\ &\quad -(\nabla h)(Y,\nabla_X Z, W) - (\nabla h)(Y,Z,\nabla_X W)\end{aligned}$$

and we have the Ricci identity

$$\begin{aligned}(1.12) \quad (\nabla^2 h)(X,Y,Z,W) - (\nabla^2 h)(Y,X,Z,W) &= R^{\perp}(X,Y)h(Z,W) \\ &\quad - h(R(X,Y)Z,W) - h(Z,R(X,Y)W)\end{aligned}$$

for any $X, Y, Z, W \in \Gamma(TM^{\perp})$.

On the other hand, since Kaehlerian submanifolds are minimal we have

$$(1.13) \qquad \sum_i h(e_i,e_i) = \sum_i (\nabla h)(X,e_i,e_i) = \sum_i (\nabla^2 h)(X,Y,e_i,e_i) = 0.$$

Now define $f : M \to \mathbb{R}, f = \frac{1}{2}\|h\|^2$; then, since $\int_M \triangle f dV = 0$, from the Ricci identity, (1.7) and (1.13), we have the following lemma:

Lemma 1. *Let M be a compact Kaehler submanifold of a complex space form. Then we have*

$$\int_M \{\|\nabla h\|^2 + \sum_{ijk}\left[R^{\perp}(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) - R((e_k, e_i, e_j, A_{h(e_i,e_j)}e_k)\right]$$
$$+\sum_{ijk}\left[R^{\perp}(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) - R((e_k, e_i, e_j, A_{h(e_i,e_j)}e_k)\right]$$
$$+\sum_{ij} Ric(e_i, A_{h(e_i,e_j)}e_i)\}dV = 0$$

The proof is exactly the same with Lemma2.1 in [1], so we omit it here.

2. A Theorem on Kaehlerian Submanifolds in a Complex Space Form

Let M be an n-dimensional compact Kaehlerian submanifold of an $(n+m)$-dimensional complex space form $\overline{M}(c)$. We choose a local orthonormal frame $\{e_1, ..., e_n, e_1^* = Je_1, ..., e_n^* = Je_n\}$ on M and a frame $\{N_1, ..., N_m, N_1^* = JN_1, ..., N_m^* = JN_m\}$ of normals, and define the function $K^{\perp} : M \to \mathbb{R}$ by

$$K^{\perp} = \sum_{ij\alpha\beta}\left[R^{\perp}(e_i, e_j, N_\alpha, N_\beta\right]^2,$$

the normal curvature of the Kaehlerian submanifold. We also define a function $\varphi : M \to \mathbb{R}$ as in [1] by

$$\varphi = 2\sum_{\alpha<\beta}\|A_\alpha\|^2\|A_\beta\|^2,$$

where $A_\alpha = A_{N_\alpha}$.

Theorem 2.1. *Let M be an n-dimensional compact Kaehlerian submanifold of the complex projective space CP^{n+m}. If $4r \geq c(4n^2 + n + 1)$ and $\sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2 \leq \varphi$ then M is totally geodesic.*

Proof. From equation (1.6) we have

$$\begin{aligned}
\sum_{ijk} R(e_k, e_i, e_j, A_{h(e_i,e_j)}e_k) &= \frac{1}{4}c\sum_{ijk}[g(e_i, e_j)g(e_k, A_{h(e_i,e_j)}e_k) \\
&\quad -g(e_k, e_j)g(e_i, A_{h(e_i,e_j)}e_k) \\
&\quad +g(Je_i, e_j)g(Je_k, A_{h(e_i,e_j)}e_k) \\
&\quad -g(Je_k, e_j)g(Je_i, A_{h(e_i,e_j)}e_k) \\
&\quad +2g(Je_i, e_k)g(Je_j, A_{h(e_i,e_j)}e_k)] \\
&\quad +\sum_{ijk} g(A_{h(e_i,e_j)}e_k, A_{h(e_i,e_j)}e_k) \\
&\quad -g(A_{h(e_i,e_j)}e_k, A_{h(e_k,e_j)}e_i) \\
&= \frac{1}{4}c\sum_{ijk}[-mg(e_i, A_{h(e_i,e_j)}e_j) \\
&\quad -g(Je_k, e_j)g(Je_i, A_{h(e_i,e_j)}e_k) \\
&\quad +2g(Je_i, e_k)g(Je_j, A_{h(e_i,e_j)}e_k)] \\
&\quad +\|A_h\|^2 - g(A_{h(e_i,e_j)}e_k, A_{h(e_k,e_j)}e_i)
\end{aligned}$$

or

$$\begin{aligned}
\sum_{ijk} R(e_k, e_i, e_j, A_{h(e_i,e_j)}e_k) &= \frac{1}{4}c\sum_{ijk}[-mg(h(e_i, e_j), h(e_i, e_j)) \\
&\quad -g(Je_k, e_j)g(h(Je_i, e_k), h(e_i, e_j)) \\
&\quad +2g(Je_i, e_k)g(h(Je_j, e_k), h(e_i, e_j))] \\
&\quad +\|A_h\|^2 - g(A_{h(e_i,e_j)}e_k, A_{h(e_k,e_j)}e_i).
\end{aligned} \tag{2.1}$$

On the other hand, we find that

$$\begin{aligned}
2\sum_{ijk} & g(Je_i, e_k)g(h(Je_j, e_k), h(e_i, e_j)) \\
&= 2\sum_{ijk\alpha} g(Je_i, e_k)g(h(Je_j, e_k), g(h(e_i, e_j), N_\alpha)N_\alpha) \\
&= 2\sum_{ijk\alpha} g(Je_i, e_k)g(h(Je_j, e_k), N_\alpha)g(h(e_i, e_j), N_\alpha) \\
&= 2\sum_{ijk\alpha} g(Je_i, e_k)g(A_\alpha e_k, Je_j)g(A_\alpha e_i, e_j)
\end{aligned}$$

$$= 2\sum_{ij\alpha}\left(\sum_k g(g(e_k, A_\alpha J e_j)e_k, Je_i\right) g(A_\alpha e_i, e_j)$$
$$= 2\sum_{ij\alpha} g(A_\alpha J e_j, Je_i) g(A_\alpha e_i, e_j).$$

Now, using the second identity of (1.5), we get

$$\begin{aligned} 2\sum_{ijk} g(Je_i, e_k) g(h(Je_j, e_k), h(e_i, e_j)) &= -2\sum_{ij\alpha} g(A_\alpha e_j, e_i) g(A_\alpha e_i, e_j) \\ &= -2\|h\|^2 . \end{aligned} \tag{2.2}$$

Similarly we derive

$$\sum_{ijk} g(Je_k, e_j) g(h(Je_i, e_k), h(e_i, e_j)) = -\|h\|^2 . \tag{2.3}$$

On the other hand,

$$\sum_{ijk} g(A_{h(e_i,e_j)}e_k, A_{h(e_k,e_j)}e_i) = \sum_{ik\alpha\beta} g(A_\alpha e_i, A_\beta e_k) g(A_\alpha e_k, A_\beta e_i). \tag{2.4}$$

Using (2.2),(2.3) and (2.4) in (2.1) we obtain

$$\begin{aligned} \sum_{ijk} R(e_k, e_i, e_j, A_{h(e_i,e_j)}e_k) &= -\frac{c}{4}(n+1)\|h\|^2 + \|A_h\|^2 \\ &\quad -\sum_{ik\alpha\beta} g(A_\alpha e_i, A_\beta e_k) g(A_\alpha e_k, A_\beta e_i). \end{aligned} \tag{2.5}$$

Taking account of equation (1.9) we have

$$\begin{aligned} \sum_{ij} Ric(e_i, A_{h(e_i,e_j)}e_j) &= \sum_{ij\alpha} g(h(e_i, e_j)N_\alpha,)Ric(e_i, A_\alpha e_j) \\ &= \sum_{j\alpha} Ric(A_\alpha e_j, A_\alpha e_j) \end{aligned}$$

and

$$\sum_{j\alpha} Ric(A_\alpha e_j, A_\alpha e_j) = \frac{1}{2}(n+1)c\|h\|^2 - \sum_{\alpha\beta}\|A_\alpha A_\beta\|^2 . \tag{2.6}$$

Thus, from (2.5) and (2.6), we get

$$(2.7)\qquad -\sum_{ijk} R((e_k, e_i, e_j, A_{h(e_i,e_j)}e_k) + \sum_{ij} Ric(e_i, A_{h(e_i,e_j)}e_i) = \frac{3c}{4}(n+1)\|h\|^2 - \|A_h\|^2 + \sum_{ik\alpha\beta} g(A_\alpha e_i, A_\beta e_k) g(A_\alpha e_k, A_\beta e_i) - \sum_{\alpha\beta} \|A_\alpha A_\beta\|^2 .$$

Now, since A is self-adjoint, from equation (1.8),

$$\begin{aligned}\sum_{ijk} R^\perp(e_k, e_i, h(e_j, e_k), h(e_i, e_j) &= \sum_{ijk} - g(\left[A_{h(e_i,e_j)}, A_{h(e_j,e_k)}\right] e_k, e_i) \\ &\quad + \frac{1}{2} c g(e_k, Je_j) g(Jh(e_j, e_k), h(e_i, e_j)) \\ &= \sum_{ijk} - g(A_{h(e_j,e_k)}e_k, A_{h(e_i,e_j)}e_i) + g(A_{h(e_i,e_j)}e_k, A_{h(e_j,e_k)}e_i) \\ &\quad + \frac{1}{2} c g(e_k, je_j) g(Jh(e_j, e_k), h(e_i, e_j)) \\ &= \sum_{ijk} - g(A_{h(e_j,e_k)}e_k, A_{h(e_i,e_j)}e_i) + g(A_{h(e_i,e_j)}e_k, A_{h(e_j,e_k)}e_i) \\ &\quad + \frac{c}{2} g(e_k, Je_j) \sum_\alpha g(Jh(e_j, e_k), g(h(e_i, e_j), N_\alpha) N_\alpha).\end{aligned}$$

From the first equality of (1.5) and (1.3) we have

$$\sum_{ijk} R^\perp(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) = \sum_{ijk} - g(A_{h(e_j,e_k)}e_k, A_{h(e_i,e_j)}e_i) + g(A_{h(e_i,e_j)}e_k, A_{h(e_j,e_k)}e_i) + \frac{c}{2} g(e_k, Je_j) \sum_\alpha g(A_\alpha e_i, e_j) g(A_\alpha e_k, Je_j)$$

or

$$\sum_{ijk} R^\perp(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) = \sum_{ijk} - g(A_{h(e_j,e_k)}e_k, A_{h(e_i,e_j)}e_i) + g(A_{h(e_i,e_j)}e_k, A_{h(e_j,e_k)}e_i) + \frac{c}{2} \sum_\alpha g(A_\alpha e_i, e_j) \sum_k g(g(e_k, A_\alpha Je_j)e_k, Je_j).$$

Hence

$$\sum_{ijk} R^{\perp}(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) = \sum_{ijk} - g(A_{h(e_j,e_k)}e_k, A_{h(e_i,e_j)}e_i) + g(A_{h(e_i,e_j)}e_k, A_{h(e_j,e_k)}e_i) + \frac{c}{2}\sum_{\alpha} g(A_\alpha e_i, e_j)g(A_\alpha Je_j, Je_j).$$

From (1.5) we derive

$$\sum_{ijk} R^{\perp}(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) = \sum_{ijk} - g(A_{h(e_j,e_k)}e_k, A_{h(e_i,e_j)}e_i) + g(A_{h(e_i,e_j)}e_k, A_{h(e_j,e_k)}e_i) - \frac{c}{2}\|h\|^2.$$

Using (2.4) in this equation we arive at

$$\begin{aligned}\sum_{ijk} R^{\perp}(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) &= \sum_{ijk} - g(A_{h(e_j,e_k)}e_k, A_{h(e_i,e_j)}e_i) \\ &\quad + \sum_{ik\alpha\beta} g(A_\alpha e_i, A_\beta e_k)g(A_\alpha e_k, A_\beta e_i) - \frac{1}{2}c\|h\|^2 \\ &= \sum_{\alpha\beta} - \|A_\beta A_\alpha\|^2 \\ &\quad + \sum_{ik\alpha\beta} g(A_\alpha e_i, A_\beta e_k)g(A_\alpha e_k, A_\beta e_i) - \frac{1}{2}c\|h\|^2.\end{aligned} \tag{2.8}$$

Moreover, since

$$\begin{aligned}\sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2 &= \sum_{ik\alpha\beta} g((A_\alpha A_\beta - A_\beta A_\alpha)e_i, e_k)^2 \\ &= -2\sum_{ik\alpha\beta} g(A_\alpha e_i, A_\beta e_k)g(A_\alpha e_k, A_\beta e_i) \\ &\quad +2\|A_\alpha A_\beta\|^2\end{aligned} \tag{2.9}$$

we arrive at

$$\sum_{ijk} R^{\perp}(e_k, e_i, h(e_j, e_k), h(e_i, e_j)) = -\frac{1}{2}\sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2 - \frac{1}{2}c\|h\|^2. \tag{2.10}$$

of (2.8)-(2.10), we obtain

$$K^{\perp}=\sum_{\alpha\beta}\|A_\alpha A_\beta-A_\beta A_\alpha\|^2+2c\|h\|^2+\frac{1}{4}c^2nm. \tag{2.11}$$

Thus using (2.11) in (2.10) we get

$$\sum_{ijk}R^{\perp}(e_k,e_i,h(e_j,e_k),h(e_i,e_j))=-\frac{1}{2}K^{\perp}+\frac{1}{2}c\|h\|^2+\frac{1}{8}c^2nm. \tag{2.12}$$

Now, using (2.9) in (2.7), we have

$$-\sum_{ijk}R((e_k,e_i,e_j,A_{h(e_i,e_j)}e_k)+\sum_{ij}Ric(e_i,A_{h(e_i,e_j)}e_i)=$$
$$\frac{3c}{4}(n+1)\|h\|^2-\|A_h\|^2-\frac{1}{2}\sum_{\alpha\beta}\|A_\alpha A_\beta-A_\beta A_\alpha\|^2.$$

Substituing (2.11) into this equation,

$$-\sum_{ijk}R((e_k,e_i,e_j,A_{h(e_i,e_j)}e_k)+\sum_{ij}Ric(e_i,A_{h(e_i,e_j)}e_i)= \tag{2.13}$$
$$\frac{c}{4}(3n+7)\|h\|^2-\|A_h\|^2-\frac{1}{2}K^{\perp}+\frac{1}{8}c^2nm.$$

From (2.12) and (2.13) we derive

$$-\sum_{ijk}R((e_k,e_i,e_j,A_{h(e_i,e_j)}e_k)+\sum_{ij}Ric(e_i,A_{h(e_i,e_j)}e_i)+$$
$$\sum_{ijk}R^{\perp}(e_k,e_i,h(e_j,e_k),h(e_i,e_j))=\frac{c}{4}(3n+9)\|h\|^2-\|A_h\|^2-K^{\perp}+\frac{1}{4}c^2nm.$$

On the other hand, since

$$\|A_h\|^2=\|h\|^4-2\sum_{\alpha<\beta}\|A_\alpha\|^2\|A_\beta\|^2,$$

we get

$$\int_M\{\|\nabla h\|^2+\|h\|^2\left(\frac{(3n+9)}{4}c-\|h\|^2\right)+\varphi-K^{\perp}+\frac{1}{4}c^2nm\}dV=0.$$

Thus, from (2.11), we have

$$\int_M \{\|\nabla h\|^2 + \|h\|^2\left(\frac{3n+1}{4}c - \|h\|^2\right) + \varphi - \sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2\}dV = 0. \tag{2.14}$$

Hence if $4r > c(4n^2+n-1)$ and $\sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2 \leq \varphi$, then $\|h\|^2 \leq \frac{(3n+1)}{4}c$. Thus, from (2.14), we have $\|h\|^2 = 0$, that is, $h = 0$.

3. Some Corollaries

From (2.11) we have the following corollaries:

Corollary 1. *Let M be a Kaehlerian submanifold of a complex projective space. Then we have*

$$K^\perp \geq \sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2 + \frac{1}{4}c^2nm$$

The equality holds if and only if M is totally geodesic.

Corollary 2. *Let M be a Kaehlerian submanifold of a complex hyperbolic space. Then we have*

$$K^\perp \leq \sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2 + \frac{1}{4}c^2nm.$$

The equality holds if and only if M is totally geodesic.

Combining corollary 1 and Corollary 2 we can give the following corollary

Corollary 3. *Let M be a Kaehlerian submanifold of a complex space form. Then M is totally geodesic if and only if*

$$K^\perp = \sum_{\alpha\beta}\|A_\alpha A_\beta - A_\beta A_\alpha\|^2 + \frac{1}{4}c^2nm.$$

From (2.12) we have the following corollaries:

Corollary 4. *There exist no Kaehlerian submanifold of a complex projective space with flat normal connection.*

Proof. We suppose that M be a Kaehlerian submanifold with flat normal connection. Then from (2.12) we have

$$\begin{aligned} \frac{1}{2}c\left\|h\right\|^2 + \frac{1}{8}c^2nm &= 0 \\ \left\|h\right\|^2 &= -\frac{1}{4}cnm \end{aligned}$$

which is impossible.

Corollary 5. *Let M be a Kaehlerian submanifold of a complex space form $(c \leq 0)$ with flat normal connection. If M is totally geodesic, then $c = 0$, that is $\overline{M}(c)$ is a complex Euclidean space.*

Proof. From (2.12) , if M is a Kaehlerian submanifold with flat normal connection, we have

$$c\left\|h\right\|^2 = -\frac{1}{4}c^2nm.$$

Hence $h = 0 \Rightarrow c = 0$.

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